(DIRECT STRESS & STRAIN) Lect. 1, 2 & 3

Axial force OR Normal Force :-

When a force is applied along any axis, an equal and opposite reaction develops along the same axis, the force is called axial force

If Force + member will fail

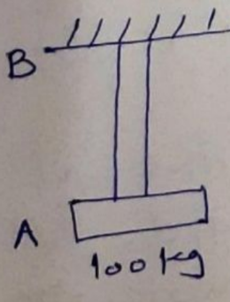
The failure surface in such case will be normal to the direction of applied force. hence also known as normal force

* The normal force either induces tension or compression in member and is known as tensile or compressive force respectively.

TENSION :-

Consider that a 100 kg wt. is suspended from a rod at its end 'A' as shown in fig. The other end B of rod is fixed at the ceiling.

For stability, support B should be strong enough so that rod that does not come out of ceiling. Hence, the system will be stable only if support B provide equal & opposite

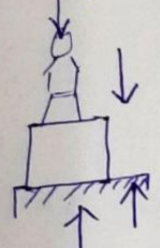


reaction to the applied load as shown. As these forces i.e action at end A and reaction at end B, are pulling apart, the rod is under tension and magnitude of tensile force is 100 kg.

Effect :- Increase in length or dimensions in the direction of load

Compression :~

Let us consider a small concrete pedestal over which a statue is placed. The wt. of statue acts downwards whereas equal & opposite reaction is provided by ground as shown in fig.



In this case, the applied load & its reaction compresses the Pedestal from end hence, applied load is called as compressive load.

Effect :- Reduction in dimension in the direction of load

Concept of stress :~

Strength is defined in terms of maximum load which an element can sustain without failure. For ex. steel wire of 1 mm dia fails by a pull of 60 kg. so we say strength of wire is 60 kg.

If dia ↑ → Definitely failure load ↑

In both case material is same but strength is different. so failure load can't be taken std. term to define the strength.

Hence st. of material need to defined in terms of load as well as area over which it is applied. A relation between both term is generally defined as stress.

→ I. Resistance developed by the body per unit cross sectional area $\boxed{\sigma = \frac{P}{A} \frac{N}{mm^2}}$

Axial OR Normal stress :~

$\rightarrow \sigma = \frac{P}{A} = \frac{\text{Load}}{\text{c/s area normal to load.}}$

→ If Applied load is pull type, cause extension in dia. ....

Similarly

$$\sigma_t = \frac{P(t)}{A} \qquad \sigma_c = \frac{P_{(c)}}{A}$$

Unit of stress :~

$1\ kg = 9.81\ N$, $\sigma = \frac{P}{A}$ or N/mm^2

$1\ N/m^2 = 1\ Pa$ (Pascal)

$1\ N/mm^2 = 10^6\ N/m^2$

$= 10^6 \times Pa$

$= 1\ MPa$ (Mega Pascal)

$1\ GPa = 10^9$ Pascal

$= 10^3 \times 10^6\ Pa$

$= 1000\ MPa$

$= 1000\ N/mm^2$

Deformation Due to Applied Load

Depending upon the type of actions a structural member may elongate, shorten bend or twist.

- Axial deformation in form of elongation or shortening
- Bending " represent by deflection & change in curvature

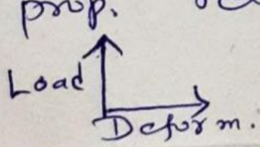
Axial Deformation Parameter :-

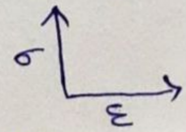
Axial deformation changes with change in original dimension of specimen. ∴ Axial deformation should be defined in a term, which relates deformation as well as original dimensions.

Axial strain ε is defined as ratio of change in dimensions to original dim-

$\varepsilon = \Delta D / D$, Where D is dimensions in the direction of external Applied forces.

When a steel rod is tested under axial Tension load. the def. (Δ or δ) remains prop. to the applied load during initial stage of loading as shown in fig.(i) If the loads & resulting def. are represented in terms of stress & strain, they also hold Same prop. relationship in fig. (ii)





$\sigma \propto \varepsilon$

$\sigma = E \cdot \varepsilon$

$E = \sigma / \varepsilon$

→ Where E is known as elastic constant & is termed as modulus of elasticity. Its value is constant for given material.

→ It is independent of shape & size.

(i) $\sigma = \frac{P}{A}$, e or $\varepsilon = \frac{\Delta l}{l}$

(ii) Putting these expression in $\sigma = E \times e$

$$\frac{P}{A} = E \times \frac{\Delta l}{l} \quad \text{or} \quad \Delta l = \frac{Pl}{AE}.$$

Ex: A steel rect. bar of c/s dim. 20 mm × 30 mm & length 3 m carries an axial Tension of magnitude 100 kN. Calculate δl. $E = 2 \times 10^5$ MPa

step. (I) Given Parameter :-

$P = 100$ kN, $b = 20$ mm, $d = 30$ mm, $l = 3$ m

$E = 2 \times 10^5$ N/mm²

To determine :- change in length (δl)

step (II) c/s area :- $A = b \times d = 20 \times 30 = 600$ mm²

step (III) :- Elongation of bar

change in length $\Delta l = \frac{P.l}{AE}$

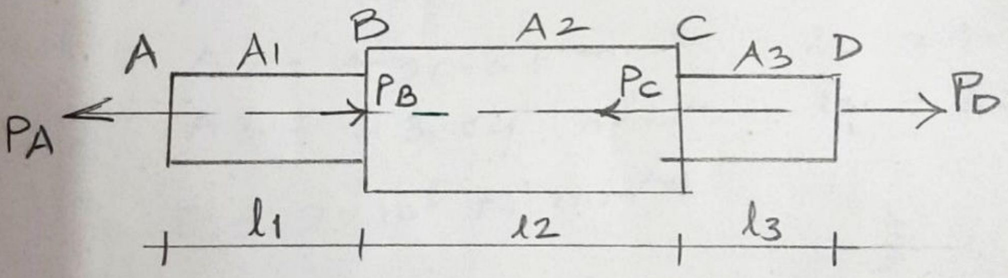
$$\boxed{\Delta l = \frac{1 \times 10^5 \times 3000}{600 \times 2 \times 10^5} = 2.5 \text{ mm}}$$

Lect.-4

BARS OF VARYING CROSS-SECTIONS.

→ Principle of superposition:-

When a member is subjected to a no. of forces acting on its outer edges as well as at some internal sections, along the length of member, the total deformation of the member will be equal to the algebric sum of the deformation of individual sections. This is true within elastic limit.



$$\delta l = \pm \delta l_1 \pm \delta l_2 \pm \delta l_3$$

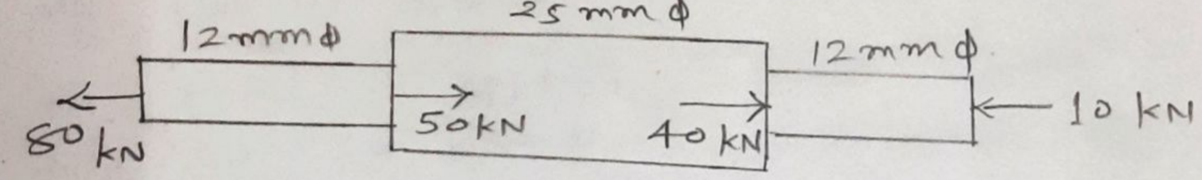
$$\therefore \delta l = \pm \frac{P_1 l_1}{A_1 E_1} \pm \frac{P_2 l_2}{A_2 E_2} \pm \frac{P_3 l_3}{A_3 E_3} \pm \ldots$$

P_1, P_2, P_3 .. are forces on section

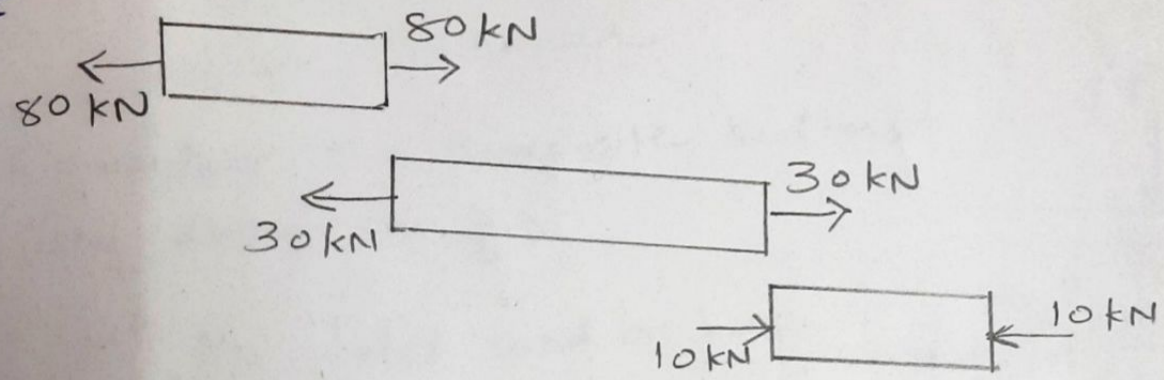
l_1, l_2, l_3 -- are length of

A_1 A_2 A_3 are area of

$E_1 = E_2 = E_3 = E$ is & the modulus of elasticity of the bar x: Calculate change in length of a bar as shown in fig. Take $E = 2\times10^5\ N/mm^2$.



ep. I



Step: 2

$A_1 = \pi/4 \times 12^2\ mm^2$ $\quad l_1 = 300\ mm$

$A_2 = 490.87\ mm^2$ $\quad l_2 = 400\ mm$

$A_3 = 113.09\ mm^2$ $\quad l_3 = 300\ mm$

$E = 2\times10^5\ N/mm^2$

$$\delta l = \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2} + \frac{P_3 l_3}{A_3 E_3}$$

$$= \frac{1}{E}\left[\frac{80\times10^3\times300}{113.09} + \frac{30\times10^3\times400}{490.87} - \frac{10\times10^3\times300}{113.09}\right]$$

$$= \frac{1}{2\times10^5}\left[210{,}139.2\right]$$

$$\boxed{\delta l = 1.05\ mm}$$

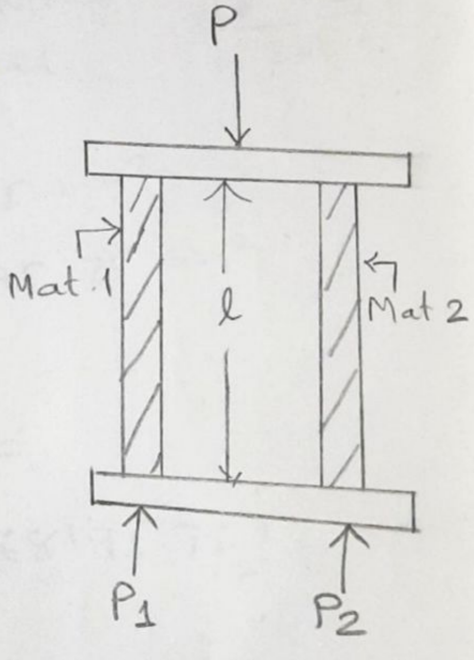
(Lect.-5)

Composite section (Lect. 5)

If a bar is made up of two or more materials, it is called –

ex. steel & copper,
" & brass,
copper & aluminium
conc. + steel

Consider the composite sections as shown in fig.



Let P = Total load on bar

P_1 = Load shared by bar 1

P_2 = " " by bar 2

A_1 = c/s area of bar 1

A_2 = " -2

E_1 = Mod. of Ela bar-1

E_2 = " -2

For composite bar elongation or contraction of each bar is same

$e_1 = e_2$ and $l_1 = l_2$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \quad \therefore \boxed{\sigma_1 = \frac{E_1}{E_2} \cdot \sigma_2} \quad \text{— (1)}$$

→ E_1/E_2 is known as modular ratio.

$E_1 > E_2$

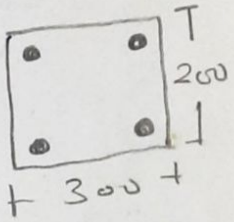
Now Total Load = load on mat. 1 + load on mat 2

$$P = P_1 + P_2$$

$$P = \sigma_1 \cdot A_1 + \sigma_2 \cdot A_2 \qquad \Big|\ \because \sigma = \frac{P}{A}$$

Ex: An R.C.C. column 230 X 300 mm is rein. with 4 no. of 16 mm dia steel bars. if σ_{cp} & σ_{sc} are 5 N/mm² & 190 N/mm². Find total load carried by column

Step. I Data given



size of col^m. (230 x 300) mm

$\sigma_c = 5 \text{ N/mm}^2$, $\sigma_s = 190 \text{ N/mm}^2$

4 no. 16 mm steel

step: 2 $A_s = \pi/4 \times (16)^2 \times 4 = 804.25 \text{ mm}^2$

$A_c = 230 \times 300 = 68195.75 \text{ mm}^2$

step: 3 $P = \sigma_s \cdot A_s + \sigma_c \cdot A_c$

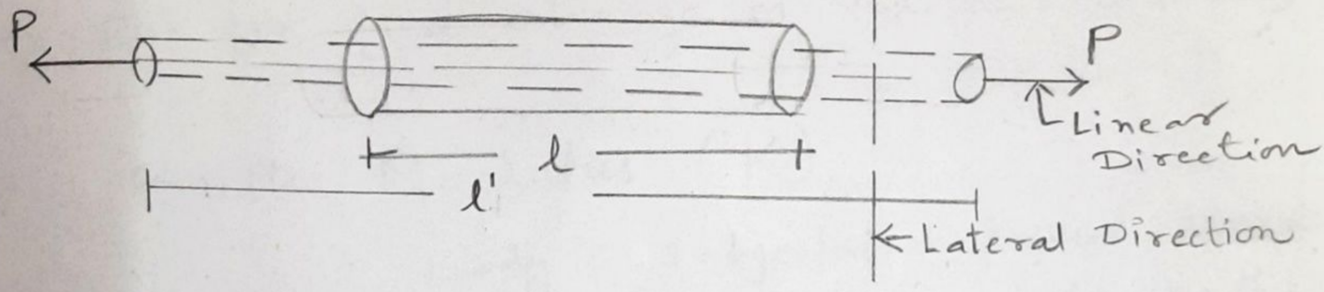
$= (190 \cdot 804.25) + (5 \times 68195.75)$

$= 493786.25 \text{ N}$

$= 493.78 \text{ kN}.$

Elastic Constants.

Consider a bar subjected to tensile force 'P' as shown in fig.



length ↑ from l to l'

The direction of force is called linear direct-

The direction of ⊥ to linear direction is lateral direct-

length of bar is a linear Direction

Diameter of " lateral ".

Linear strain $\varepsilon = \frac{\delta l}{l}$, It has no unit

For Rect. c/s. $\varepsilon' = \frac{\delta b}{b}$ or $\varepsilon' = \frac{\delta t}{t}$

Poisson's ratio :- $\frac{\text{Lateral strain}}{\text{linear strain}}$

$\frac{1}{m}$ or (μ)

For steel $\mu = 0.25$ to 0.33

For conc $\mu = 0.08$ to 0.18

→ Volumetric strain (δv)

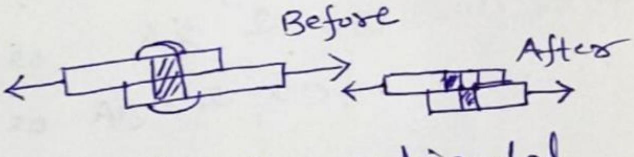
$$\delta v = \delta v / v$$

→ Eq. for find change in vol $\frac{\delta v}{v} = \varepsilon(1-2\mu)$

→ Bulk Modulus: (K)

Body subjected to equal three mutually ⊥ stresses of equal intensity, the ratio of direct stress to corresponding vol. strain is Bulk Modulus.

$$K = \frac{\sigma}{\delta v} \text{ -- } N/mm^2$$

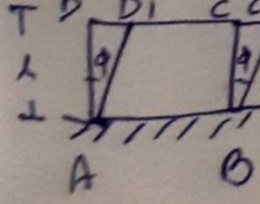


→ Shear stress :-

When a body is subjected to two equal & opposite forces acting tangentially across the resisting sections, as a result of which the body tends to shear off across the section, the stress induced is called shear stress.

$$\tau = F/A_s \text{ -- } N/mm^2$$

Shear strain :- consider a cube of length l fixed at bottom face AB.

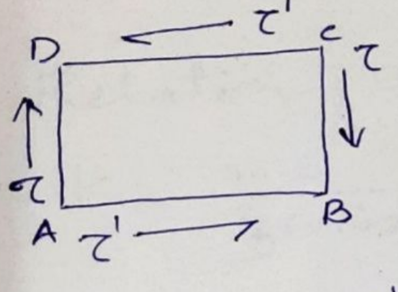


→ P is to face DC

→ Result :- cube ABCD → ABC_1D_1

$$\text{shear strain } (\phi) = \frac{\text{Deformation}}{\text{original length}} = \frac{CC_1}{l}$$

Complimentary Shear stress



In order to cause an eq. shear stress (τ). across a plane, is always accompained by balancing shear stress (τ') across the plane & normal to it. The balancing shear stress is called C...

→ Block ABCD. shear stress (τ) on faces AD & CB.

$P = \tau \times AD = \tau \times CB$.

∴ This forces will form a couple.

Moment of couple = Force × dist^n.

$M_1 = (\tau \times AD) \times AB$. —(i)

If the block is in eq^n. resisting couple with shear stress τ' on faces AB & CD will set up.

∴ forces acting on faces AB & CD.

$P = \tau' AB = \tau' \times CD$

This forces will also form a couple.

Moment of this couple

$M_2 = (\tau' \times AB) \times AD$ — (ii)

eq^n: two $M_1 = M_2$

$\tau \times AD \times (AB) = (\tau' \times AB) \times AD$

$\boxed{\tau = \tau'}$

τ = shear stress

τ' = complimentary shear stress.

Modulus of Rigidity (C, G or N)

C = shear stress / shear strain

→ Relation Betn E & K :-

$$K = \frac{m \cdot E}{3(m-2)} \qquad \frac{1}{m} = P.R$$

→ Relation Betn E & G :-

$$G = \frac{m \cdot E}{2(m+1)}$$

→ Relation Betn. E, G & K.

$$E = \frac{9GK}{G+3K}$$

X:1 A specimen has G 6×10^4 N/mm² & E 1.5×10^5 N
Find μ of material.

Step. I) Data given:

E = —   G = —

Find μ

Step. II

we know

$$G = \frac{m \cdot E}{2(m+1)}$$

$$6 \times 10^4 = \frac{m \times 1.5 \times 10^5}{2(m+1)}$$

$$12 \times 10^4 m + 12 \times 10^4 = 1.5 \times 10^5 m$$

$$30{,}000 m = 12 \times 10^4$$

$$\mu = \frac{1}{m} = \frac{1}{4} = \boxed{0.25}$$

Derive eq^n of strain energy for gradual load

In case of gradual load system,
load gradually increases from 0 to 'P'

$\therefore$ average load $= \frac{0+P}{2} = \frac{P}{2}$

Now

$$W.D. = \text{avg. load} \times \text{deformation}$$

$$= \frac{P}{2} \times \delta l$$

$$= \frac{\sigma . A}{2} \times e.l$$

$$= \frac{1}{2} . \sigma . e . Al$$

$$= \frac{1}{2} . \sigma . \frac{\sigma}{E} . V$$

$$\boxed{U = \frac{1}{2} . \frac{\sigma^2}{E} . V}$$

$\because \sigma = P/A$

$e = \delta l / l$

$E = \sigma / e$

$V = A . l$.

$\therefore$ A rect. bar 60 mm wide & 40 mm th. 1 m length is subjected to axial Tensile P 50 kN. E = 2× 1.5×10^5 N/mm^2 Find amount of S.E.

step: 1 Data given

① $b \times d = (60 \times 40)$ mm , $P = 50$ kN (gradual)

$l = 1000$ mm $\quad E = 1.5 \times 10^5$ N/mm^2

For gradual load,

$$\sigma = \frac{P}{A} = \frac{50 \times 10^3}{60 \times 40} = 20.83 \text{ N/mm}^2$$

$$vol = A \times l = 60 \times 40 \times 1000 =$$

Now $S.E. = \frac{\sigma^2}{2E} \times V = \frac{(20.83)^2}{2 \times 1.5 \times 10^5} \times (60 \times 10 \times 1000)$

$$\boxed{u = 347.11 \text{ N.mm}}$$

Lect. 7

Strain Energy & Impact Loading.

→ When a body is strained within elastic limit, energy stored in it. This energy is called .....

strain energy = Work done on a body.

$$\boxed{u = \frac{\sigma^2}{2E} \times V}$$

Where u = strain energy
σ = stress
V = Vol. of body

unit of S.E N.m.

2. Resilience :-
Total S.E in body, within elastic limit...

$$\boxed{R = u = \sigma^2/2E \times V}$$

3. Proof resilience :- Total S.E. that can be stored in a body at elastic limit --

$$\boxed{U_p = \frac{(\sigma_E)^2}{2E} \times V}$$

4. Modulus of Resilience (U_m)
The max. S.E. that can be stored in a body per unit volume at elastic is .....

$$\boxed{U_m = \frac{\frac{\sigma_E^2}{2E} \times V}{V} = \frac{(\sigma_E)^2}{2E}}$$

The unit of Mod. of Res. is $N.mm/mm^3$

5. Instantaneous stress:

When a body is subjected to sudden load or impact load, the stress induced is ....

6. Methods of applying load :-

- Gradual Loading
- sudden loading
- Impact Loading
- Shock Loading

• Gradual Load :- $\sigma = P/A$

• Sudden Load :- $2P/A$

• Impact Load :- Load fall on a body from some height ....

$$\sigma = \frac{P}{A}\left[1 + \sqrt{1 + \frac{2EAh}{P.l}}\right]$$

strain energy is

$u =$ work done

$u = P(h + \delta l)$

$$\boxed{\frac{\sigma^2}{2E} \times V = P(h + \delta l)}$$ — eqn for find impact load

where σ = stress due to impact Load

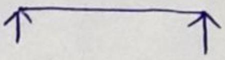
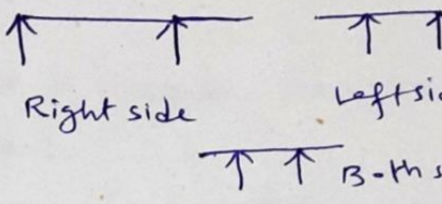
P = Impact load

h = ht. of fall of load

δl = elongation of body

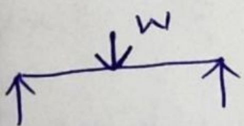
SHEAR FORCE & BENDING MOMENT

* Types of Beams

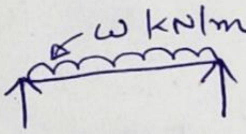
1. Simply supported Beam 
2. Overhanging Beam 

3. Cantilever 
4. Fixed 
5. Continuous 

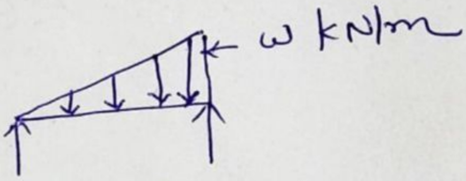
* Types of Loads



Point



U.d.l



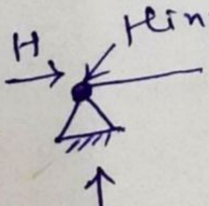
u.v.l

* Types of Supports :-

(1) 

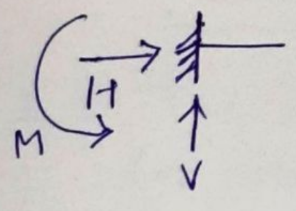
Simple support

simply supported on support
no monolithic construction
betn beam & support
(only Vertical R. develop)

(2) Hinge 

Beam can rotate @ Hinge
V & H. develop.

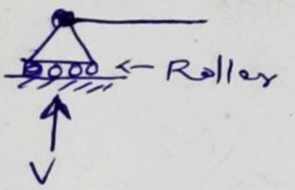
(3) Fixed Support :-



Reactions. H, V, M

end is rigidly fixed or built wal

(4) Roller Support :-



End of beam can move o rollers.

only V Reaction to plane

- Provided for bridge girde to allow free expa. & cont.

→ Shear force (SF)

Algebric sum of unbalanced vertica forces to the Left or right side of section..

unit N or kN

→ Bending Moment :-

Algebric sum of moments to the left or right side of sections is ....

unit N.m or kN.m.

→ Point of Contraflexure :-

The Pt. in a B.M. dia at which B.M. changes sign from +ve or -ve or -ve to +ve. ....

At this B.M. is zero.

Imp for P.C. (Point of inflection or Virtual hinge)

1. Beam can't resist B.M
2. one side of P.C. curvature of beam is sagging & other side Hogging.

Relation Betn S.F. & B.M.

1. The rate of change of S.F. w.r.t. distance (or slope of S.F. curve) = to the intensity of loading
   $\therefore \delta F / \delta x = W$ — (i)

2. The rate of change of B.M. w.r.t. distance (or slope of B.M. curve) = to the S.F. at section
   $\therefore \delta M / \delta x = -F$ — (ii)

3. The point at which S.F. changes sign, B.M. Max

S.F. Diagram :-

All diffn sections of beam values of S.F. are calculated. The Positive values are plotted above the base line & negative " " " below " ".

A curve or straight line obtained by joining these points is ----

B.M. Diagram :-

At different sections of beam values of B.M. are calculated.

+ values above the Base line
— " below " ".

A curve or st. line obtained by joining these all pts. is ...

* The points To be kept in Mind While Drawing S.F. & B.M. Diagram.

(i) For S.F. Dia

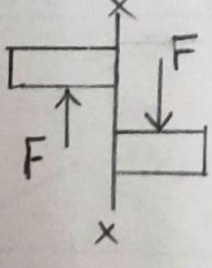
1. No load, represted by hori. line i.e. S.F. constant
2. Point load, " Vert. line.
3 U.d.l. " inclined line.

(ii) For B.M dia

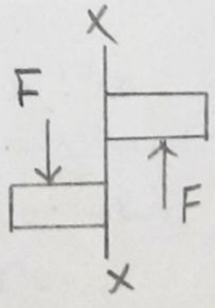
1. Free end of cantilever B.M. is zero.
2 Both ends of S.S. " "
3 No load, " st. line
4. Point ~~load~~, at which S.F. change sign, B.M. Max.
5 U.d.l. is acting, B.M. will be parabolic curve

→ Sign Conventions :-

(a) For S.F. :-

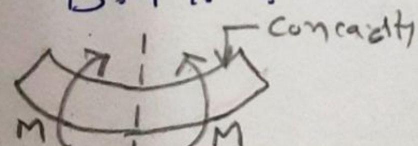


+ve S.F.

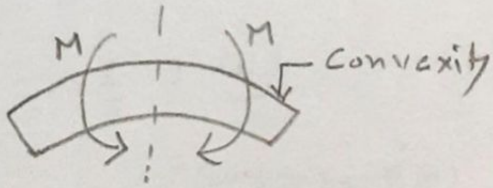


-ve S.F.

(b) For B.M. :-



+ve B.M.
(Sagging)



-ve B.M.
(Hogging)

Sagging B.M.

→ Tendency of Beam to bend the upside with concavity at top.

→ taken +ve

Hogging B.M

to bend downside with convexity at top

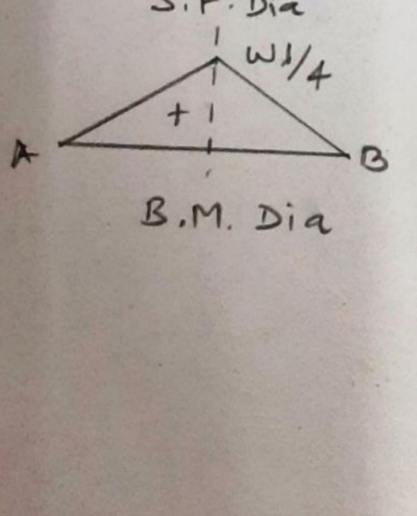
taken −ve

Importance of Sagging & Hogging B.M.

1. In case of Sagging B.M. Tension at Bottom & Comp. at Top, steel is placed at Bot.
   e.g. S.S. Beam

2. In case of Hogging B.M. Tension at Top & comp. at Bot. Steel is placed at Top.
   e.g. Cantilever Beam

→ S.S. Beam with Point load at Mid pt.



- $RA = RB = W/2$
- S.F. cal^n:
  SF at A $= W/2$
  S.F at C $= \frac{W}{2} - W = -\frac{W}{2}$
  SF at B $= -W/2$
- B.M. cal^n:
  $MA = 0$
  $MB = 0$
  $MC = \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$

(Sagging BM)

Lect. 1

Moment OF Inertia

The moment of force about any point is defined as product & ⊥ dist^n bet^n direction of force & point under consideration. It is also called first moment of force.

In fact moment doesnot necessary involve force them, a moment of any other physical term can also be determined simply by multiplying magnitude of physical quantity & ⊥ dist.

ex. Moment = Area × ⊥ dist^n.

If moment of moment is taken about same reference axis, it is known as moment of inertia in terms of area. which is

$$I_A = (M \times y) = A \cdot y \times y = A \cdot y^2$$

where I_A is area of Moment of inertia A is area & 'y' is the dist^n. bet^n centroid of area & reference axis. M of I is also determined on similar notes, in terms of mass.

$$I_m = mr^2$$

where m = mass

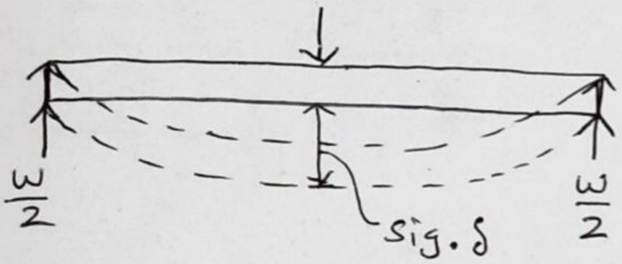
r = dist^n bet^n centre of mass & ref. axis

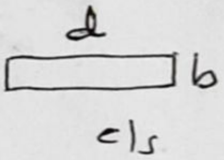
→ M. I will change with change in location of reference axis.

Importance of M.I.

The material of structural element (like beam, column), its c/s, the arrangement of c/s with respect to loads etc are very imp. in the design of structural elements.

Case. 1

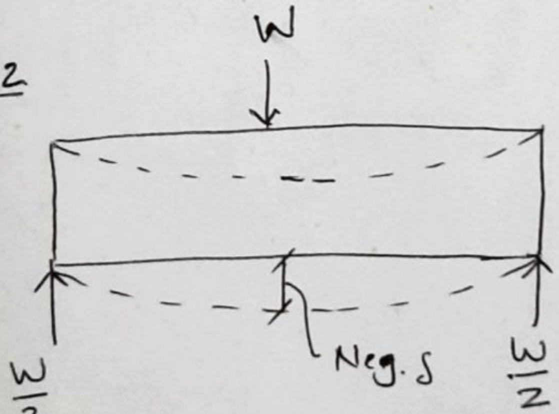




$$I_1 = \frac{db^3}{12}$$

b is in bending plane. less mat. to resist bending.
∴ δ More

Case. 2

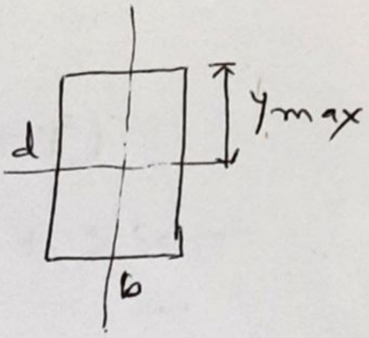


$$I_2 = \frac{bd^3}{12}$$

$$I_2 > I_1$$

d is in bending plane
~~c/s~~ ~~mo~~ Beam material more to resist bending
∴ δ will less.

→ Sectional Modulus. (Z)



$$Z = \frac{I}{y_{max}}$$

For □ sect.

$$Z_{xx} = \frac{I_{xx}}{d/2}$$

$$Z_{yy} = \frac{I_{yy}}{b/2}$$, unit mm^3 or cm^3

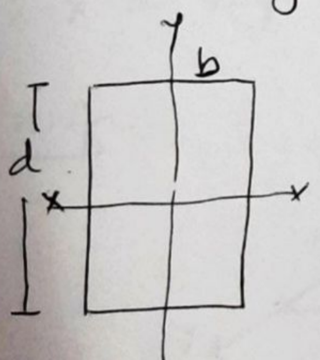
→ Radius of Gyration (k)

$$k = \sqrt{\frac{I}{A}}$$

Imp. parameter which includes effect of C/A as well as M. of I.

→ M. I of some std. sections.

1) Rectangular Section



$$I_{xx} = \frac{bd^3}{12}$$

$$I_{yy} = \frac{db^3}{12}$$

$$Z_{xx} = \frac{I_{xx}}{y_{max}} = \frac{\frac{bd^3}{12}}{d/2} = \frac{bd^2}{6}$$

$$Z_{yy} = \frac{I_{yy}}{y_{max}} = \frac{\frac{db^3}{12}}{b/2} = \frac{db^2}{6}$$

$$I_{AB} = \Sigma \delta a (h+y^2)$$

$$= \Sigma \delta a (h^2 + 2hy + y^2)$$

$$= \Sigma h^2 \delta a + \Sigma 2hy \delta a + \Sigma \delta a \cdot y^2$$

$$= ah^2 + 0 + I_g$$

$\because$ $\Sigma \delta a y^2 = I_g$

$\Sigma h^2 \delta a = ah^2$

$\Sigma day = a\bar{y} = 0$

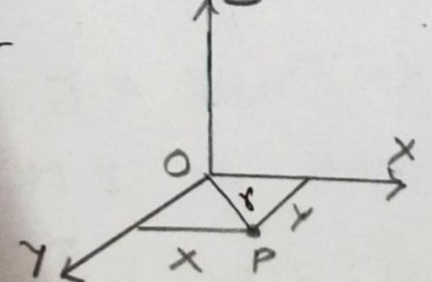
$\bar{y} = 0$

$$\boxed{\therefore \quad I_{AB} = I_g + ah^2}$$

→ Perpendicular Axis Theorem

If I_{xx} & I_{yy} be the moments of inertia of a plane sections @ two ⊥ axes meeting at O, the M.I. @ Z-Z axis, ⊥ to the plane & passing th. the intersection of X-X & Y-Y axes given by $\boxed{I_{zz} = I_{xx} + I_{yy}}$

Proof :-



consider a small (P) of area δa

x & y are the co-ordinates of P along OX & OY.

$\therefore r^2 = x^2 + y^2$

OP = r.

M. I. of P @ X-X axis,

$I_{xx} = \delta a \cdot y^2$, Similar $I_{yy} = \delta a \cdot x^2$.

$I_{zz} = \delta a \cdot r^2 = \delta a (x^2 + y^2) = \delta a x^2 + \delta a \cdot y^2$

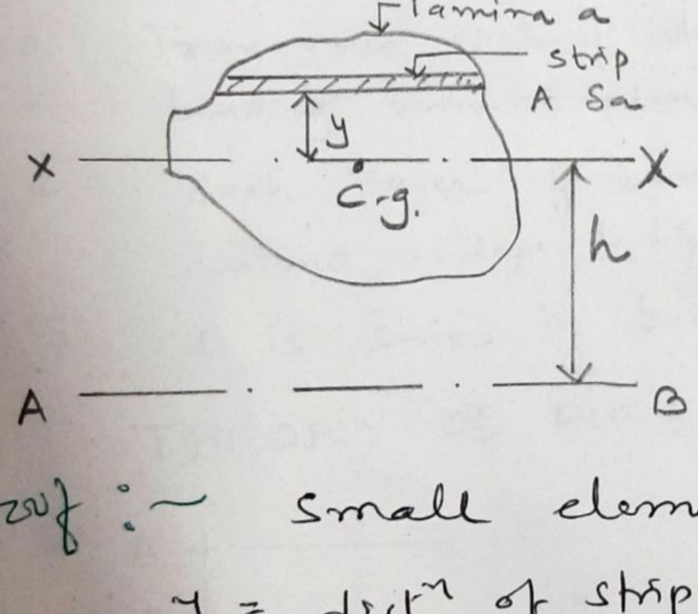
$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Parallel Axis Theorem :-

If I_g is the moment of inertia of a plane area about an axis passing through its centre of gravity, then M.I. of the area about axis AB, ‖ to the first axis, and at a distⁿ h from centre of gravity is given by :

$$\boxed{I_{AB} = I_g + Ah^2}$$

Where
I_{AB} = M.I. of area @ AB
I_g = " " @ c.g.
a = area of section
h = distⁿ betⁿ c.g. & axis AB.



Proof :- Small elemental strip of area δa

y = distⁿ of strip of c.g. of sect.

∴ M.I. of strip @ c.g. of sect.

$$I = \delta a \cdot y^2$$

M.I. of whole sect. about axis passing th. c.g.

$$I_g = \Sigma da \cdot y^2$$

∴ M.I. of whole section @ axis AB

5. BENDING STRESSES IN BEAM Lect 1

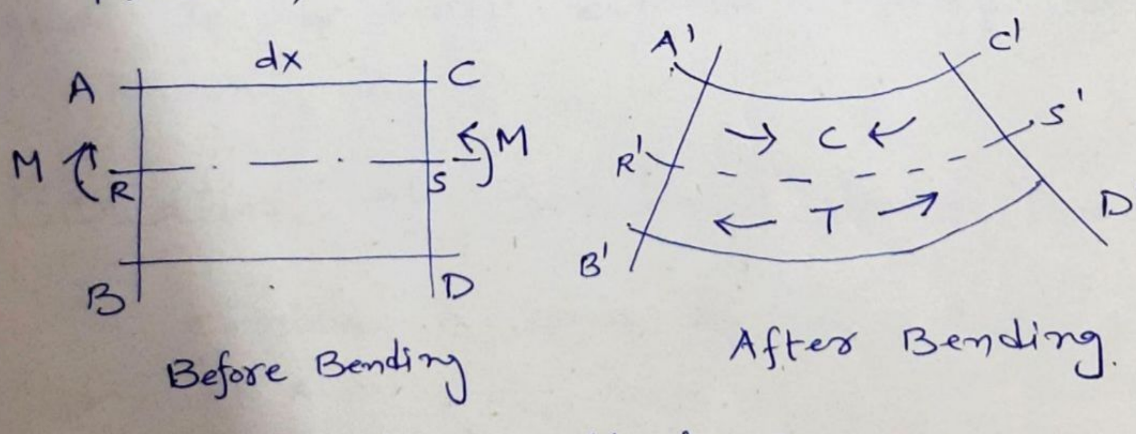
5.1 Pure Bending stress :-

When a beam length is subjected to a constant amount of bending Moment & a zero shear force, the stresses set up across the c/s of the beam due to bending is ..

5.2 **Assumption in the Theory of Pure Bending.**

1. material - perfectly homogenous. & isotropic
2. Beam is stresses within its elastic limit. & Hook's law is valid
3. Transverse sections, which were plane before bending remain plane after bending.
4. Each layer of beam is free to expand or contract., independently.
5. E is same in tension & compression

5.3 THEORY OF PURE BENDING :-



Before Bending

After Bending.

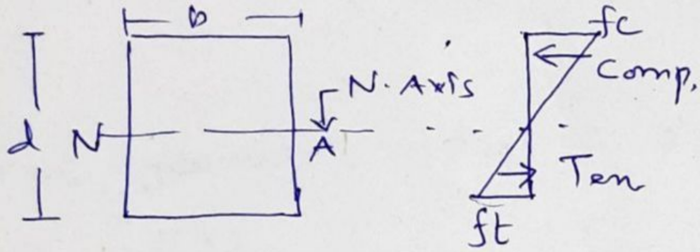
→ Beam S.S., dx = small length of beam

Due to Bending top layer AC suffered A'C' - Comp.
BD " B'D - Tens.

• Neutral layer (Neutral plane).

Due to Bending, the layers above RS comp. & below stretched. But RS neither comp. nor stretched. is ...

• Neutral Axis :- The line of intersection of the neutral layer, with any normal c/s of a beam is .. At this axis no stress of any kind



• Moment of Resistance :-

When a beam length is subjected to constant amount of bending moment, on one side of N.A, comp. stresses & on other tensile stresses. These stresses form a couple, whose moment must be equal to or resist external Bending Moment is known as --

→ Equation of Bending stress. (Flexure eqn)

consider a layer PQ at a dist^n. y from N.A.

After bending, PQ to P'Q' (comp.)

∴ Decrease in length of PQ layer

$\delta l = PQ - P'Q'$

$$\text{Strain} = e = \frac{\delta l}{\text{original length}} = \frac{PQ - P'Q'}{PQ} \quad \text{—(i)}$$

Now, from geometry, $OP'Q'$ & $O'R'S'$ are similar.

$$\therefore \frac{P'Q'}{R'S'} = \frac{R-y}{R}$$

$$\therefore \quad 1 = \frac{P'Q'}{R'S'} = 1 - \frac{(R-y)}{R}$$

$$\therefore \frac{R'S' \cdot P'Q'}{R'S'} = \frac{R-R+y}{R}$$

$$\therefore \frac{R'S' - P'Q'}{R'S'} = \frac{y}{R} \qquad \Big| \because R'S' = PS = PQ = \text{N. layer.}$$

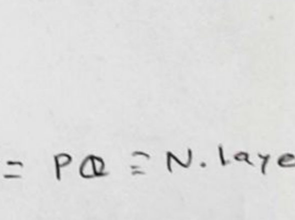
$$\therefore \frac{PQ - P'Q'}{PQ} = \frac{y}{R} \quad \text{—(ii)}$$

$$\therefore \quad e = \frac{y}{R} \qquad \because \text{bending stress } (f) = e \times E$$

$$e = f/E$$

$$\therefore \frac{f}{y} = \frac{E}{R} \quad \text{— (iii)}$$

Now a small layer PQ of a Beam section at a dist^n y from NA



from eq^n (iii); $f = y \cdot \frac{E}{R}$

$\therefore$ Total force in this layer $= f \times \delta a$

$= y \times \frac{E}{R} \times \delta a$

$\therefore$ Moment @ N.A.

$= (y \times \frac{E}{R} \times \delta a) \times y = \frac{E}{R} \cdot y^2 \cdot \delta a$

$\therefore$ Algebric sum of all such Moments @ NA

$M = \Sigma \frac{E}{R} \cdot y^2 \cdot \delta a = \frac{E}{R} \Sigma y^2 \cdot \delta a$

But $\Sigma y^2, \ \delta a = 1$

$M = E/R \cdot 1 \qquad \therefore \frac{M}{I} = \frac{E}{R}$ — (iv)

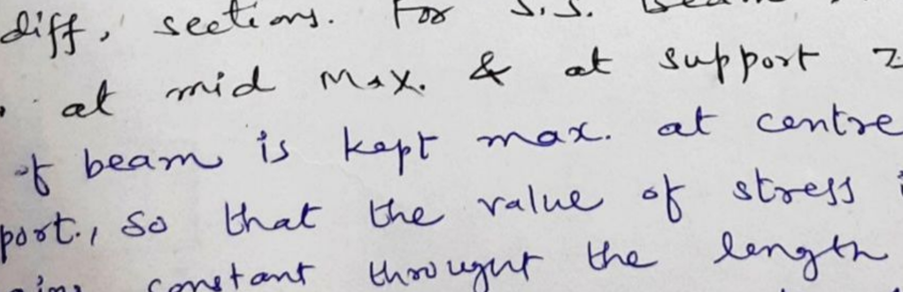
$\therefore$ from eq^n (iii) & (iv)

$$\boxed{\frac{M}{I} = \frac{f}{y} = \frac{E}{R}}$$ eq^n for Flexure e--

Beam OF Uniform Strength :-

When a beam is subjected to external load, the value of B.M. will be diff. at diff. sections. For S.S. Beam having U.d.l. B.M. at mid Max. & at support zero. If the c/s of beam is kept max. at centre & mini. at support, so that the value of stress in beam remains constant throught the length such set---

Benfit :- Economical use of material.

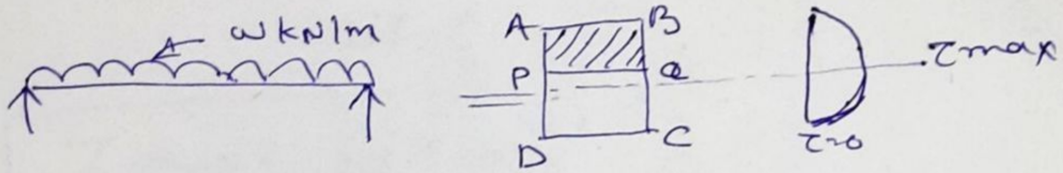


In case of cantilever Beam.. it differs.

6. SHEAR STRESSES IN BEAMS.

When a beam is subjected to external load, S.F. and B.M. is produced in beam. stress produced in beam to resist S.F. is called shear stress. stress produced to resist bending Moment is called Bending stress.

→ Eq^n For shear stress.



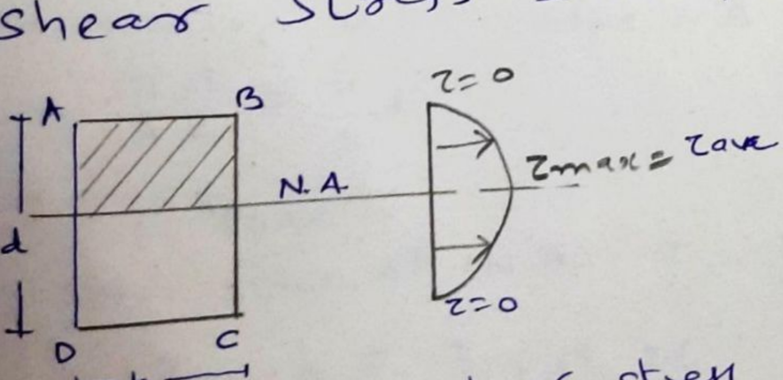
$$\tau = \frac{F \cdot A \cdot \bar{y}}{I \cdot b}$$

where F = S.F.
A = c/s area of section above or below the PQ.
$\bar{y}$ = distⁿ. of c.g. of area from NA
b = width of section.

→ Average shear stress :-

τ_{avg} = shear stress / shear Area

→ shear stress Dist. For Rect. section



(a) c/s of beam

(b) shear stress Dist. Dia

consider a rectangular section of width b & depth d as shown in fig

$A = b \times d/2$ (A = area of section above N.A)

y = dist^n of c.g. of area above ...

$= \frac{d/2}{2} = \frac{d}{4}$, $I = bd^3/12$

Now shear stress at N.A

$$\tau = \frac{F A \cdot \bar{y}}{I \cdot b}$$

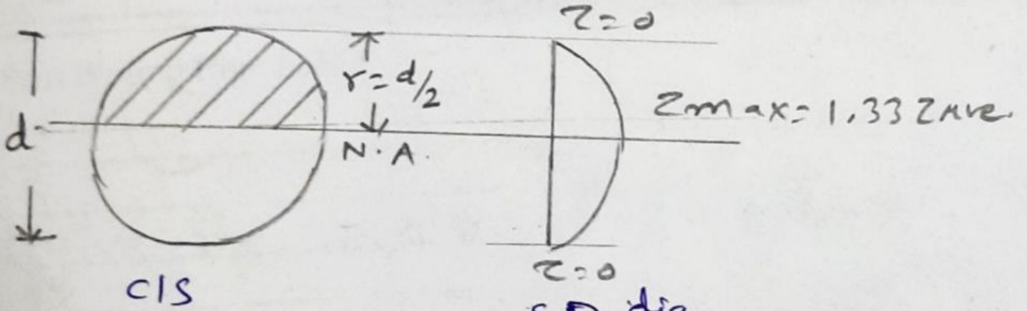
$$\therefore \tau_{max} = \frac{F \cdot b \cdot d/2 \cdot d/4}{\frac{bd^3}{12} \times b}$$

$$= \frac{3}{2} \times \frac{F}{b \cdot d} \qquad \because \tau_{ave} = \frac{F}{A}$$

$$= 1.5 \frac{F}{A}$$

$$\boxed{\tau_{max} = 1.5\, \tau_{ave}}$$

→ Shear stress Dist. For ⊙ section



C/S

S.D. dia

A = area of section above N.A., $\bar{y}$ = dist^n of c.g. of A

$A = \frac{\pi d^2}{2 \cdot 4}$ ... $= \frac{\pi}{8} d^2$ $\qquad y = \frac{4r}{3\pi} = \frac{4 \cdot d/2}{3\pi} = \frac{2d}{3\pi}$

$I = \pi/64 \times d^4$

Now shear stress at N.A. $\tau = \frac{FA\bar{y}}{Ib}$

$$\tau_{max} = \frac{F \cdot \frac{\pi}{8} \times d^2 \times \frac{2d}{3\pi}}{\pi/64 \cdot d^4 \cdot d} = \frac{F \cdot \frac{d^3}{4} \cdot \frac{1}{3}}{\frac{1}{16}\left(\frac{\pi}{4} \times d^2\right) d^3}$$

$$\tau_{max} = \frac{4F}{3\left(\frac{\pi}{4} \times d^2\right)} \qquad \tau_{ave} = \frac{F}{A}$$

$$\tau_{max} = \frac{4}{3}\frac{F}{A} = 1.33\, \tau_{avg}.$$

COLUMN AND STRUTS.

Strut :-

- A structural member sub. to axial comp. force is . .
- Vertical, horizontal or inclined
- c/s small
- Used in roof truss & bridge trusses.

Column :-

- When strut is vertical is - -.
- c/s are large
- carry heavy comp. loads.
- Used in concrete & steel buildings.

→ Difference Betn strut & column.
Ref. above notes.

→ Radius of Gyration (k)

The distn from the given axis at which, if all the elements of the lamina are placed, the M.I. of the lamina @ given axis doesnot changes. The distn ....

$$k = \sqrt{\frac{I}{A}} \quad \text{or} \quad I = Ak^2$$

Where
k = rad. of gyr.
I = M.I.
A = c/s. area

Slenderness Ratio (λ) :- $\frac{\text{eff. length of col}^n}{\text{Min. rad. of gyr}} = \frac{l_e}{k_{min}}$

If λ for col^n ↑ the load carrying cap. will be less ↓
If λ " " ↓ " more ↑

→ **Long column**

When length of column ↑ as compared to c/s dim.

$\frac{l_e}{d} \geq 15$ OR

$\lambda = \frac{l_e}{k_{min}} \geq 50$

Short column.

When l of col^n is ↓ as compared to c/s dim.

$\frac{l_e}{d} < 15$ OR

$\lambda = \frac{l_e}{k_{min}} < 50$

For mild steel if $\lambda \geq 80$, it is called long col^n.

Crushing Load :-

In short col^n with ↑ in axial comp. load, ↑ in comp. stress.

After some load it may fail by crushing.

Crippling OR Buckling OR Critical load.

with In long col^n. ↑ in axial comp. load ↑ in comp. stress.

After some load col^n starts buckling ....

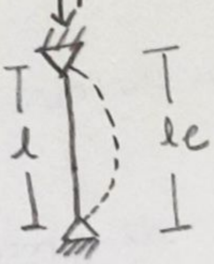
In this σ_0 is neg. ob considered.

Buckling depends on

1 Amount of load
2. length of col^n
3 End conditions
4 c/s Dim. of col^n.
5 Material of col^n.

COLUMN END CONDITIONS & EFFECTIVE LENGTH

1) Both ends hinged :~

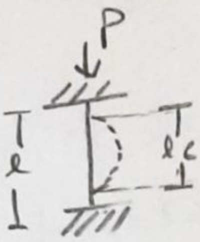


$$\boxed{l_e = l}$$

l = actual length col

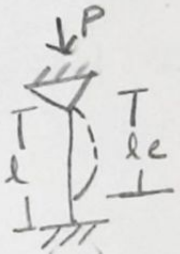
l_e = eff. length of col.

2) Both ends Fixed :~



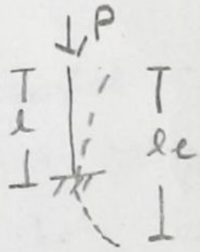
$$\boxed{l_e = \frac{l}{2}}$$

3) one end fixed, other hinged



$$\boxed{l_e = l/\sqrt{2}}$$

4) one end fixed, other free



$$\boxed{l_e = 2l}$$

→ Euler's Formula for crippling Load

$$P_E = \frac{\pi^2 EI}{(l_e)^2}$$

where

P_E – Euler's crip. load

E – Youngs Mod

I – M. I. (min)

l_e – Eff. length

→ Limitations of Euler's formula

When λ for col is < 80, the col. is considered to be short, & Euler's for. can't be applied

Assumptions of Euler's formula :-

1. long col^n
2. mat. of col^n is elastic, homo. & iso.
3. Load is truely axial.
4. c/s of col. is uniform th length
5. Hook's law is valid.
6. col^n is st. before application of load.
7. Failure of col^n is due to buckling.
8. shortening of col due to axial comp. load is neglected.

→ Rankine Formula :-

When col is sub. to axial comp. load, short col^n fails by crushing & long col^n fails by buckling. Euler's for long col^n. Rankine proposed an impirical relation which can be applied to both short & long col^n.

$$\boxed{P_R = \frac{f_c \cdot A}{1+\alpha\left(\frac{l_e}{k}\right)^2}}$$

f_c - Constant it is given
α - "

Ex. A ⊙ steel col^n having 200 mm dia fixed Both ends. Use Euler's formula. Find safe load of col^n. length of col^n 3.6 m $E = 2 \times 10^5$ N/mm^2. F.O.S = 2.

Sol^n:- Data given:

$d = 200$ mm, $l = 3.6$ m

To find P_E, P_{safe}

$I = \frac{\pi}{64} \times d^4$

$l = 3600$ mm

$= \pi/64 \times (200)^4$

F.S. = 2.

$= 78.53 \times 10^6$ mm^4.

Both ends fixed

$\therefore \; l_e = \frac{l}{2} = \frac{3600}{2} = 1800$ mm

As per Euler's formula

$$P_E = \frac{\pi^2 EI}{(l_e)^2}$$

$$= \frac{\pi^2 \cdot 2 \cdot 10^5 \cdot 78.53 \times 10^6}{(1800)^2}$$

$$= 47.843 \times 10^6 \text{ N} = 47843 \text{ kN}$$

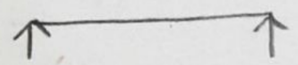
$$P_{safe} = \frac{P_E}{F.S.} = \frac{47843}{2} = \boxed{23921.5 \text{ kN}}$$

7. ANALYSIS OF SIMPLE TRUSS

- **TRUSS :-** Rigid structure composed of no. of st. member by pin joined at ends.
  → (ONLY AXIAL FORCES)

Frame :- Rigid structure composed of no. of straight member by rigidly connected at ends. (Forces may act any where on member may subjected to axial, S.F. B.M)

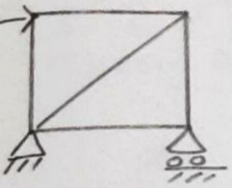
Differce Betn. Beam and Truss :-

	Beam	Truss.
1		
2	made by 1 member	made by no. of members
3	only Transverse load	loaded at joints only
4	Simple, hinged or fixed	All joints are pinned
5	Due to Transverse S.F & B.M. produced	Due to loads at joints Truss subjected to Comp & Tension.
6	For small & med. span	For large span
7	In use:- Buildings	Factry & steel bridges.
8	Repairing difficult	Easy to repair

TYPES OF TRUSSES

Perfect

$m = 2j - r$



$m = 5,\ j = 4,\ r = 3$

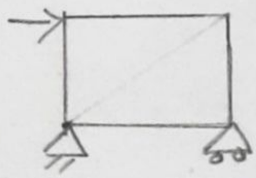
$r = 3$ ($\because$ two at hinge one at Roller)

Imperfect

$m \neq 2j - r$

Deficient truss

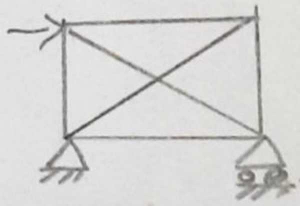
$m < 2j - r$



$m = 4,\ r = 3,\ j = 4$

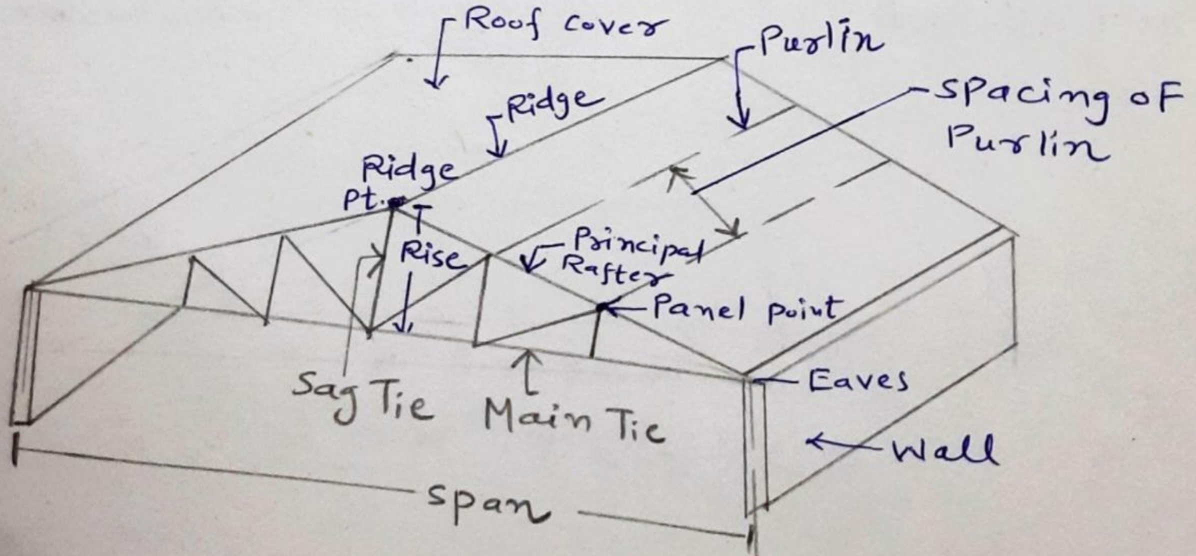
Redudant Truss

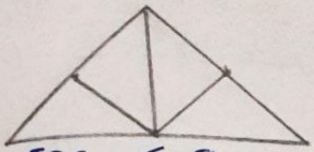
$m > 2j - r$



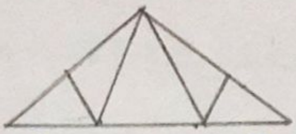
$m = 6,\ j = 4,\ r = 3$

COMPONENTS OF ROOF TRUSS

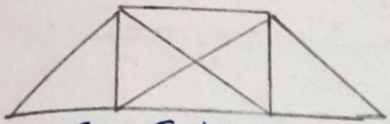




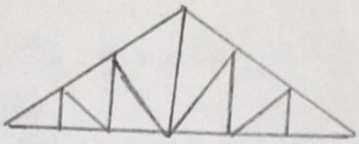
Span < 9 M
KING POST



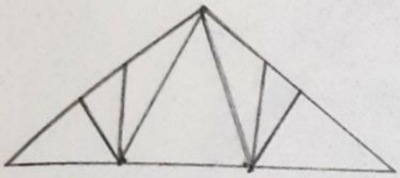
Span 6 to 9 m
FINK TYPE



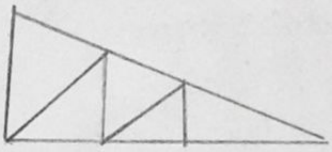
S - 6 to 9
QUEEN Post



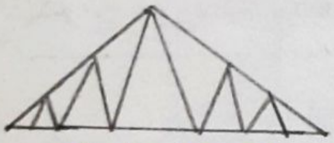
SPan 6 to 24 m
HOW TYPE



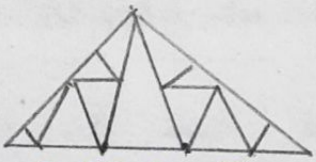
S - 9 to 12 m
FAN TYPE



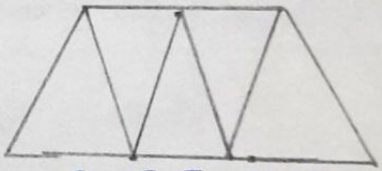
S = 5 to 8 m
NORTH LIGHT TYPE



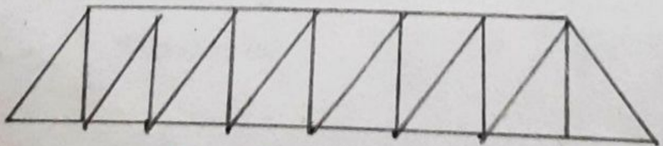
6 to 30 m
TRIANGULAR



S 12 TO 18 m
COMPOUND



S 6 TO 30 m
WARREN TYPE

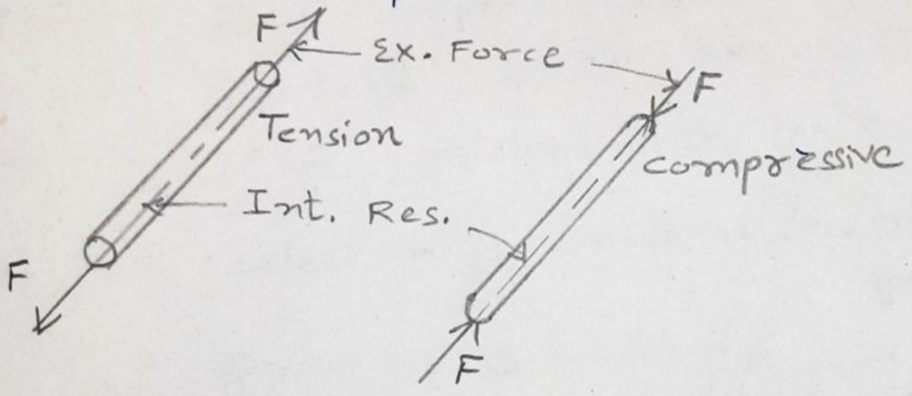


S 6 TO 24 m
HOW TYPE (FLAT)

Internal Resistance (stresses) in the Member

Tension :~ Tends to pull or stretch its joint away from its pin, ...

Compression :~ Tends to push i.e. joint towards its pin ...



Assumption Made in Analysis of Plane Truss.

1. All the joints are pinned joints.
2. The truss is loaded at the joints only.
3. The truss is a perfect truss.
4. The members are subjected to Tension or Comp
5. The members of a truss are slender & straight.
6. Self wt. of the members are neglected

Methods OF Analysis of Truss :~

- Method of Joint
- Method of Sections
- Graphical method

Method of Joint :- [STEPS]

useful in finding forces in members

step: 1 Decide by using $m = 2j - 3$
Perfect or imperfect.

step: 2 Find Support reactions by 3 conditions of eqlli. $\Sigma H = 0$, $\Sigma V = 0$, $\Sigma M = 0$
(In cantilever truss, no need to cal.)

step: 3 Select a jt. where max. two unknown force exist.

Draw F.B.D. of the selected joint.

Apply 2 conditions of eqll.
$\Sigma H = 0$, $\Sigma V = 0$, Find forces in members.

step: 4 select another jt. where only two unknown members forces exist. Find out member forces as per step. 3

Step 5 Repeat the procedure by diff. joint selections & find out forces in all members of truss.

step 6 show the nature of forces by arrow heads.

step: 7 Tabulate forces in all members as below.

Force Table

Sr. No.	Member	Force	Nature