8th Jan '18
Monday

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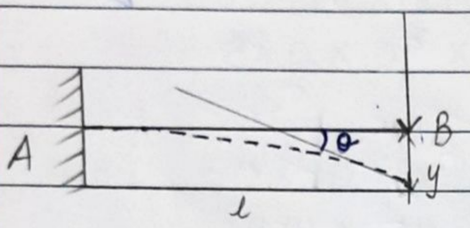
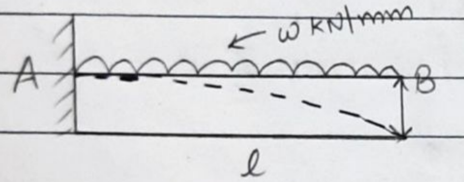
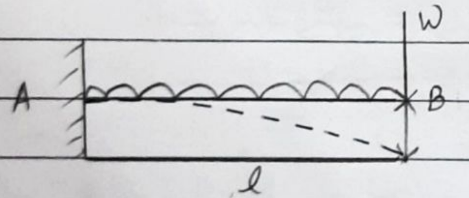
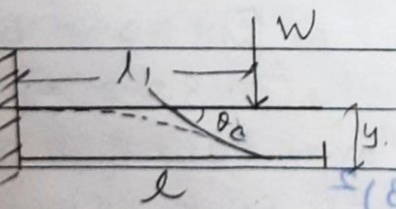
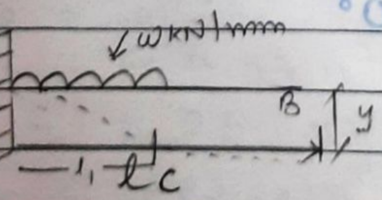
Date :

Slope And Deflection

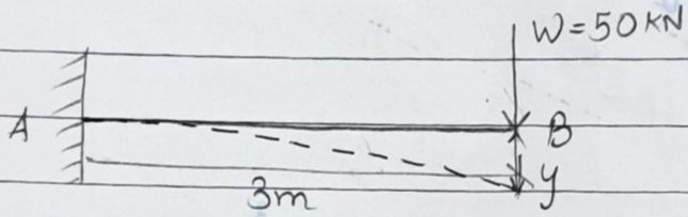
- Type of beam (Cantilever beam)

Position of load	Slope (θ)	Deflection (y)
(1) 	$\theta_B = \frac{WL^2}{2EI}$	$y_B = \frac{WL^3}{3EI}$
(2) 	$\theta_B = \frac{wL^3}{6EI}$	$y_B = \frac{wL^4}{8EI}$
(3) 	$\theta_B = \frac{WL^2}{2EI} + \frac{wL^3}{6EI}$	$y_B = \frac{wL^3}{3EI} + \frac{wL^4}{8EI}$
(4) 	$\theta_B = \frac{wL^2}{2EI} = \theta_C$	$\frac{wL^3}{3EI} + \frac{wL^2}{2EI}(l-l_1)$
(5) 	$\theta_B = \frac{wL^3}{6EI} = \theta_C$	$y_C + \theta_C (l-l_1)$ $= \frac{wL^4}{8EI} + \frac{wL^3}{6EI}(l-l_1)$

- Flexural rigidity :- It is the product of young's modulus and moment of Inertia

- Examples ...

(1)



$E = 2 \times 10^5 \, N/mm^2$

$I = 4 \times 10^8 \, mm^4$

$W = 50 \times 10^3 \, N$

$l = 3 \times 10^3 \, mm$

$\theta_B = ?$

$y_B = ?$

$$\theta_B = \frac{Wl^2}{2EI}$$

$$= \frac{50 \times 10^3 \times (3 \times 10^3)^2}{2 \times 2 \times 10^5 \times 4 \times 10^8}$$

$$= \frac{50 \times \cancel{10^3} \times 9 \times \cancel{10^6}}{4 \times \cancel{10^5} \times 4 \times \cancel{10^8}}$$

$$= \frac{450}{16 \times 10^4} = 2.81 \times 10^{-3} \text{ Rad.}$$

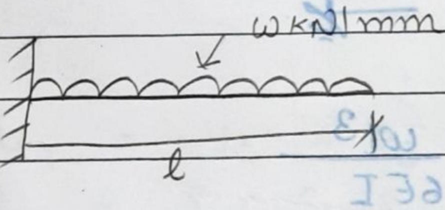
$$y_B = \frac{WL^3}{3EI}$$

$$= \frac{50 \times 10^3 \times (3 \times 10^3)^3}{3 \times 2 \times 10^5 \times 4 \times 10^8}$$

$$= \frac{50 \times 10^3 \times 2.7 \times 10^{10}}{24 \times 10^{13}}$$

$$= 5.6 \text{ mm}$$

(2)



$l = 2 \text{ M} = 2 \times 10^3 \text{ mm}$

$W = 25 \text{ kN/m}$

$E = 2.2 \times 10^5 \text{ N/mm}^2$

$I = 4 \times 10^8 \text{ mm}^4$

$$\theta_B = \frac{wL^3}{6EI}$$

$$= \frac{25 \times (2 \times 10^3)^3}{6 \times 2.2 \times 10^5 \times 4 \times 10^8}$$

$$= \frac{2 \times 10^{11}}{5.2 \times 10^{14}} = 3.79 \times 10^{-4} \text{ Radian}$$

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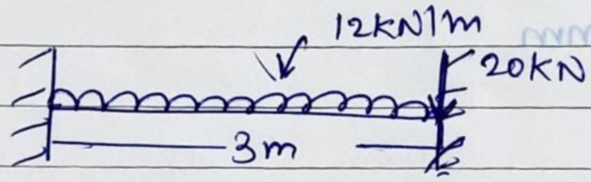
$$y_B = \frac{wL^4}{8EI}$$

$$= \frac{25 \times (2 \times 10^3)^4}{8 \times 2.2 \times 10^5 \times 4 \times 10^8}$$

$$= \frac{4 \times 10^{14}}{7.04 \times 10^{14}}$$

$$= 0.57 \text{ mm}$$



$$\theta_B = \frac{wl^2}{2EI} + \frac{wl^3}{6EI}$$

$$= \frac{20 \times 10^3 \times 3000^2}{2 \times 2 \times 10^5 \times 80 \times 10^6} + \frac{12 \times 3000^3}{6 \times 2 \times 10^5 \times 80 \times 10^6}$$

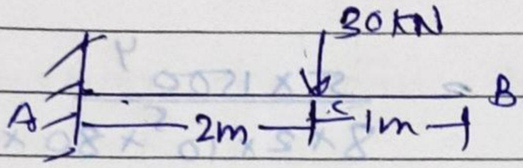
$$5.625 \times 10^{-3} + 3.375 \times 10^{-3}$$

$$= 9 \times 10^{-3} \text{ radian.}$$

$$y_B = \frac{wl^3}{3EI} + \frac{wl^4}{8EI}$$

$$= \frac{20 \times 10^3 \times 3000^3}{3 \times 2 \times 10^5 \times 80 \times 10^6} + \frac{12 \times 3000^4}{8 \times 2 \times 10^5 \times 80 \times 10^6}$$

$$= 11.25 + 7.59375 = 18.84 \text{ m.}$$



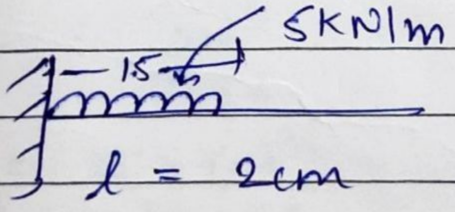
$$\theta_B = \theta_C = \frac{W l_1^2}{2EI} = \frac{30\times10^3 \times 2000^2}{2\times2\times10^5\times80\times10^6}$$

$$= 3.75 \times 10^{-3} \text{ radian}$$

$$y_C = \frac{W l_1^3}{3EI} = \frac{30\times10^3\times2000^3}{3\times2\times10^5\times80\times10^6}$$

$$= 5\text{mm}$$

$$y_B = y_C + \theta_C (l - l_1)$$
$$= 5 + 3.75\times10^{-3}(3000 - 2000)$$
$$= 5 + 3.75$$
$$= 8.75 \text{ mm}$$



$l = 2\text{m}$

$l_1 = 1.5\text{m}$

$$\theta_C = \frac{W l_1^3}{6EI} = \frac{5\times1500^3}{6\times2\times10^5\times80\times10^6}$$

$$= 1.75\times10^{-4} \text{ radian}$$

Date :

$$y_C = \frac{wl_1^4}{8EI} = \frac{5 \times 1500^4}{8 \times 2 \times 10^5 \times 80 \times 10^6}$$

$$= 0.197\text{ mm}$$

$$y_B = y_C + \theta_C (l - l_1)$$

$$= 0.197 + 1.75 \times 10^{-4} (2000 - 1500)$$

$$= 0.197 + 0.087$$

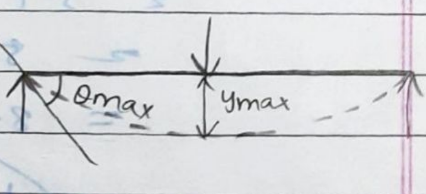
$$= 0.284\text{ mm.}$$

- Simply Support beam.

(1) Centre Point load

$$\theta_{max} = \frac{WL^2}{16EI}$$

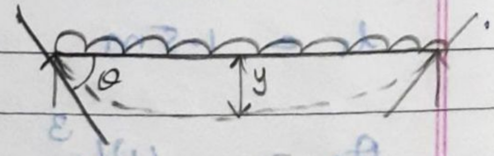
$$Y_{max} = \frac{WL^3}{48EI}$$



(2) U.d.l

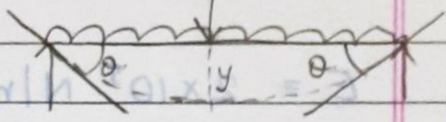
$$\theta_{max} = \frac{wl^3}{24EI}$$

$$Y_{max} = \frac{5}{384} \frac{wl^4}{EI}$$



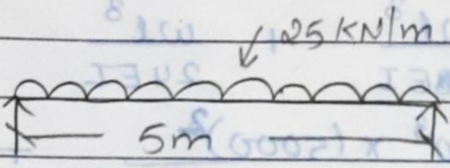
(3) Combination of point load & u.d.l.

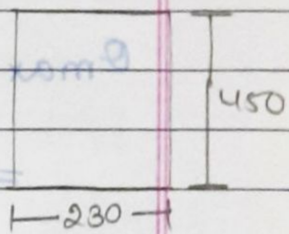
$$\theta_{max} = \frac{Wl^2}{16EI} + \frac{wl^3}{24EI}$$



$$Y_{max} = \frac{Wl^3}{48EI} + \frac{5}{384}\frac{wl^4}{EI}$$

(1)





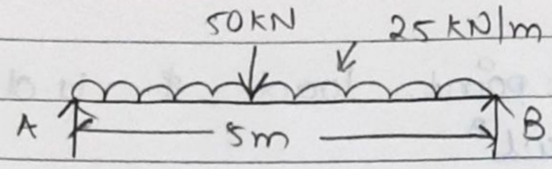
$E = 200$ GPa
$= 2 \times 10^5$ N/m.

$$I = \frac{bd^3}{12} = \frac{(230) \times (450)^3}{12}$$
$$= 1.74 \times 10^9 \; mm^4$$

$$\theta_{max} = \frac{wl^3}{24EI}$$
$$= \frac{25 \times (5000)^3}{24 \times 2 \times 10^5 \times 1.74 \times 10^9}$$
$$= \frac{3.125 \times 10^{12}}{8.352 \times 10^{15}}$$
$$= 3.74 \times 10^{-4} \text{ radian}$$

$$Y_{max} = \frac{5}{384}\frac{wl^4}{EI}$$
$$= \frac{5 \times 25 \times (5000)^4}{384 \times 2 \times 10^5 \times 1.74 \times 10^9}$$
$$= \cancel{\frac{7.8625 \times 10^{13}}{1.38 \times 10^{17}}}$$
$$= 0.584 \text{ mm}$$

(2)



$E = 2 \times 10^5 \ N/m^2$

$I = 1.74 \times 10^9 \ mm^4$

$$\theta_{max} = \frac{Wl^3}{16EI} + \frac{wl^3}{24EI}$$

$$= \frac{50 \times 10^3 \times (5000)^2}{16 \times 2 \times 10^5 \times 1.74 \times 10^9} + \frac{25 \times (5000)^3}{24 \times 2 \times 10^5 \times 1.74 \times 10^9}$$

$$= \frac{1.25 \times 10^{12}}{5.56 \times 10^{15}} + 3.74 \times 10^{-4}$$

$$= 2.24 \times 10^{-4} + 3.74 \times 10^{-4}$$

$$= 5.98 \times 10^{-4} \text{ radian}$$

$$Y_{max} = \frac{WL^3}{48EI} + \frac{5}{384} \frac{wl^4}{EI}$$

$$= \frac{50 \times 10^3 \times (5000)^3}{48 \times 2 \times 10^5 \times 1.74 \times 10^9} + \frac{5}{384} \frac{25 \times (5000)^4}{2 \times 10^5 \times 1.74 \times 10^9}$$

$$= \frac{6.25 \times 10^{15}}{1.67 \times 10^{16}} + 0.584$$

$$= 0.374 + 0.584$$

$$= 0.958 \text{ mm.}$$

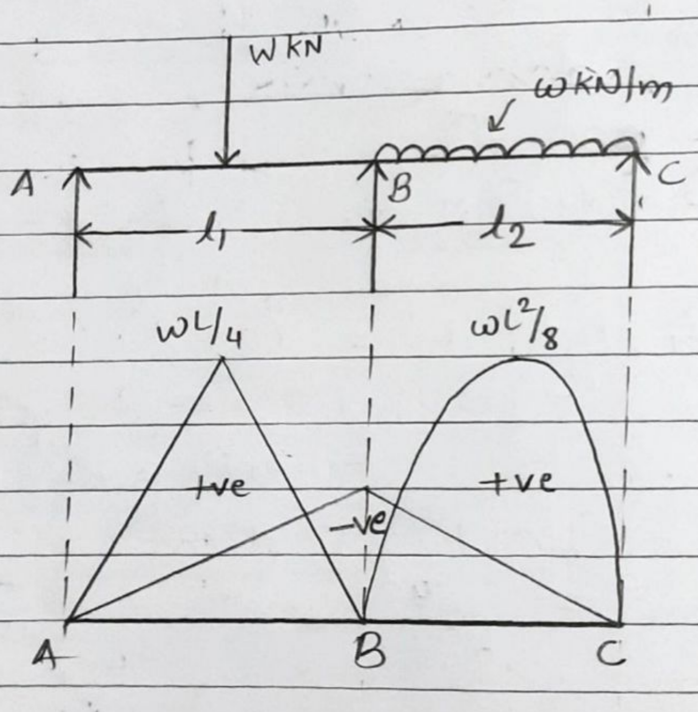
- $1^\circ = \dfrac{\pi}{180}$ Radian

19th Feb'18

Ch-3 :- CONTINUOUS BEAM

- Clayperon Theorem of 3 Moment ...

If a beam has 'n' support and end supports are fixed then the same number of equations are required to determine the support moments, which may be obtained from consecutive pairs of span i.e, AB - BC, ... etc.



$$Eq^n :- M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -\frac{6a_1x_1}{l_1} - \frac{6a_2x_2}{l_2}$$

where, M_A, M_B, M_C = Support moment

l_1 = span length

l_2 = span length

a_1 = area of M diagram for first span - AB

CLASS NOT

21/2/18

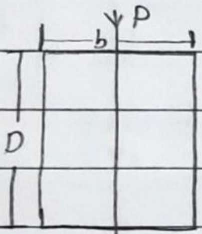
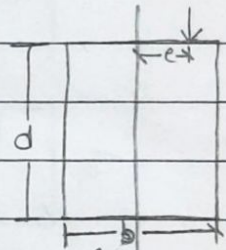
Combined Direct And Bending Str...

Direct Stress $\rightarrow \frac{F}{A}$

Bending Stress $\rightarrow \frac{M}{I} = \frac{f}{y}$ (N/mm^2) $= \frac{M}{Z} = f$

Section modulus $\rightarrow Z = \frac{I}{y}$

- Difference between Axial load and Ecentric load.

Axial load	Eccentric load.
 $\sigma_0 = \frac{P}{A}$	 $\sigma_0 = \frac{P}{A}$ + $\sigma_B = \frac{M}{Z}$
Axial load on a column	Eccentric load on a column
- When load is acting along the longitudinal axis of the column it is known as axial load.	- A load whose line of action does not coincide with the axis of a column is known as an ecentric load.
- Produces direct stress (compressive in nature) in column.	- Ecentric load produces direct stress and bending stress in a column.

Eccentricity :-

The horizontal distance between the longitudinal axis and line of action of force is known as eccentricity.

- Effect of Axial load and Eccentric load on a column.

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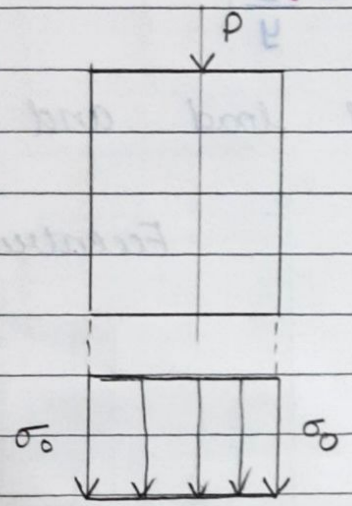
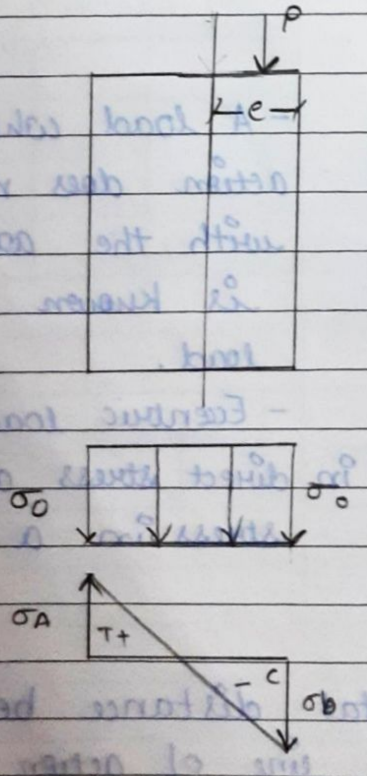
$\bullet$ C = + ↓
T = − ↑

(I)



$$\sigma_0 = \frac{P}{A}$$

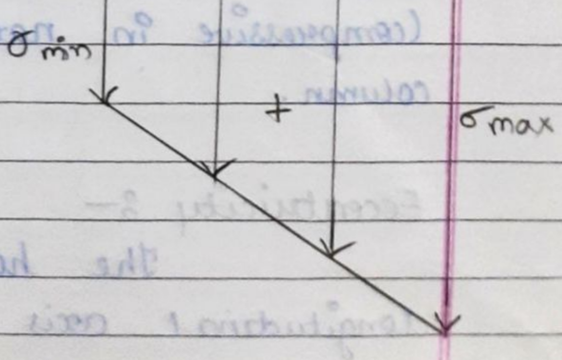
(II)



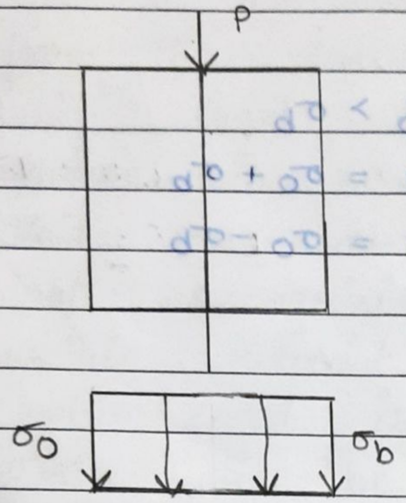
$$\sigma_0 > \sigma_b$$

$$\sigma_c = \sigma_0 + \sigma_b$$

$$\begin{cases} \sigma_c = \sigma_0 - \sigma_b = \sigma_{min} \\ \sigma_c = \sigma_0 + \sigma_b = \sigma_{max} \end{cases}$$



(III) Axial load

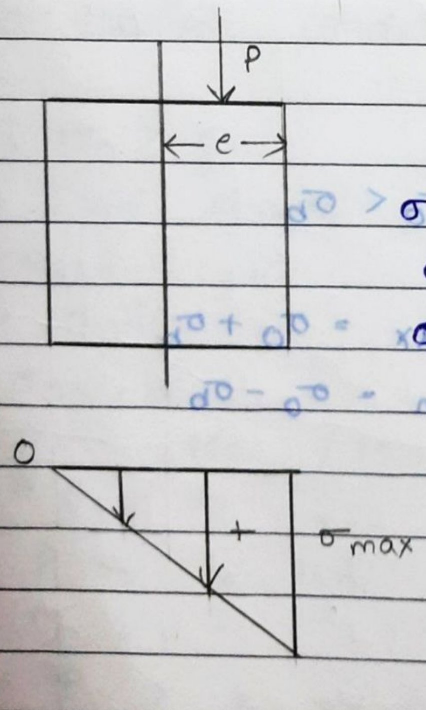


$e = 0$

$\sigma = \sigma_0 + \sigma_b$

$= \sigma_b$ (C)

(IV) Eccentric load.



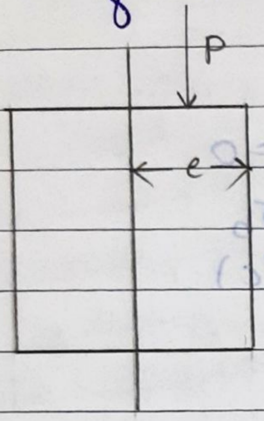
$\sigma_0 = \sigma_b$

$\sigma = \sigma_0 + \sigma_b$

$\sigma_{max} = \sigma_0 + \sigma_b$

$\sigma_{min} = \sigma_0 - \sigma_b = 0$

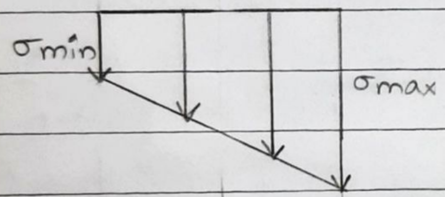
(V) Limit of eccentric

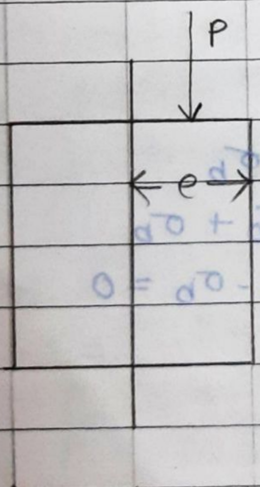


$$\sigma_0 > \sigma_b$$

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$\sigma_{min} = \sigma_0 - \sigma_b$$

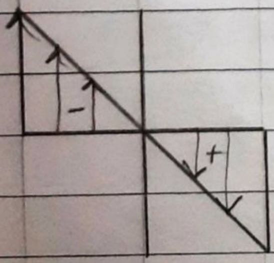




$$\sigma_0 < \sigma_b$$

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$\sigma_{min} = \sigma_0 - \sigma_b$$



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- Limit of eccentric (e limit) :-

The maximum distance of load from the center of column such that if load act within this distance there is no tension in the column. This max. distance is called as limit of eccentricity. Then load is acting within e limit σ_{min} will be compressive when load is acting at point of e limit σ_{min} will be zero. When the load is acting beyond e limit σ_{min} will be tensile.

- No tension condition :-

1) $\sigma_0 \geq \sigma_b$
2) Load should act within elimit

e limit

$\sigma_0 \geq \sigma_b$

$\frac{P}{A} \geq \frac{M}{Z}$ $(M = P \times e$

$\frac{P}{A} \geq \frac{P \times e \text{ limit}}{Z}$

$\frac{P}{A} \times \frac{Z}{P} \geq e\text{limit}$

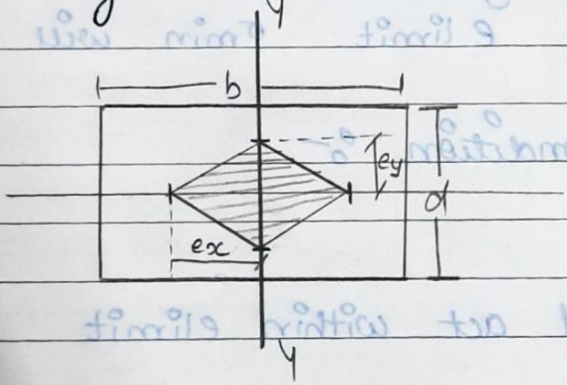
$\boxed{\frac{Z}{A} \geq e \text{ limit}}$ ... No tension condition

1/3/18

- Core or Kernel of the Section :—

The central part in the cross-section of a column joining the points of elimit such that if load acts within this part there will be no tension induced in the column. This central part is known as core or Kernel of the column.

(1) Rectangle ..



$A = b \times d$

$e = \dfrac{Z}{A}$

$Z_{yy} = \dfrac{I_{yy}}{y} = \dfrac{db^3}{12} \times \dfrac{2}{b}$

$I_{yy} = \dfrac{db^3}{12}$ $Z_{yy} = \dfrac{db^2}{6}$

$y = \dfrac{b}{2}$

$e_x = \dfrac{\dfrac{db^2}{6}}{b \times d}$

$= \dfrac{db^2}{6 \times b \times d}$

$\boxed{e_x = \dfrac{b}{6}}$

$Z_{xx} = \dfrac{I_{xx}}{y} = \dfrac{bd^3}{12} \times \dfrac{2}{d} = \dfrac{bd^2}{6}$

$I_{xx} = \dfrac{bd^3}{12}$ $y = \dfrac{d}{2}$

$e_y = \dfrac{bd^2}{6 \times b \times d}$

$\boxed{e_y = \dfrac{d}{6}}$

Hollow → ②

Eg. 2, 3, 5, 13, 16, 20

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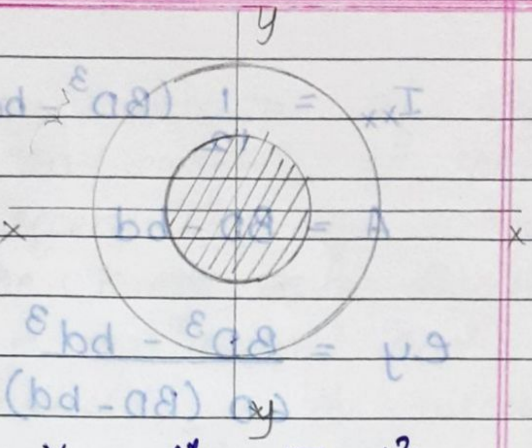
(II) Circle ..



$$e_x = e_y = \frac{Z}{A}$$

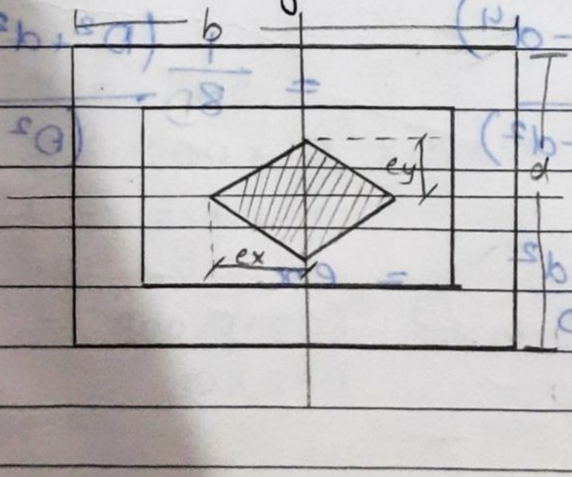
$$Z = \frac{I}{y}$$

$$I = \frac{\pi}{64} \times d^4 \qquad Y = \frac{d}{2} \qquad \frac{\pi}{4} d^2$$

$$Z = \frac{\frac{\pi}{64} \times d^4}{d/2} = \frac{\pi \times d^4 \times 2}{64 \times d} = \frac{\pi d^3}{32}$$

$$e_x = e_y = \frac{\pi \times d^3 \times 4}{32 \times \pi \times d^2} = \frac{d}{8}$$

(III) Hollow Rectangular ...



$$e = \frac{Z}{A}$$

$$e_y = \frac{Z_{xx}}{A}$$

$$Z_{xx} = \frac{I_{xx}}{y} = \frac{(BD^3 - bd^3)}{6D}$$

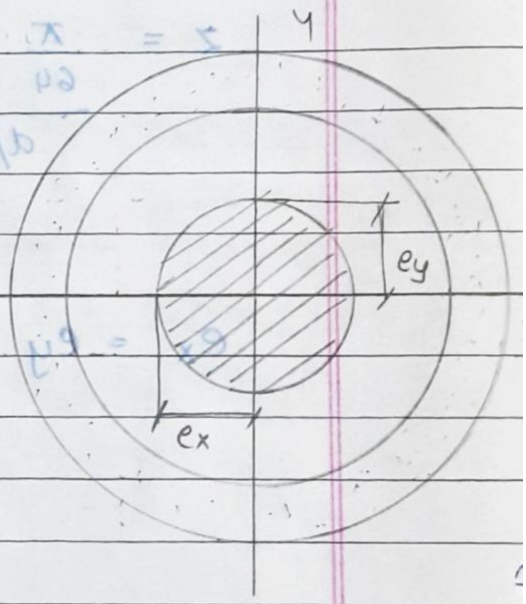
$$I_{xx} = \frac{1}{12}(BD^3 - bd^3) \qquad y = \frac{D}{2}$$

$$A = BD - bd \qquad Z_{yy} = \frac{I_{yy}}{y} = \frac{DB^3 - db^3}{6B}$$

$$e_y = \frac{BD^3 - bd^3}{6D(BD - bd)} \qquad I_{yy} = \frac{1}{12}(DB^3 - db^3)$$

$$e_x = \frac{DB^3 - db^3}{6B(BD - bd)}$$

(4) Hollow Circle :-



$$e = \frac{Z}{A} \qquad e_y = \frac{Z_{xx}}{A}$$

$$Z_{xx} = \frac{I_{xx}}{y} = \frac{\pi/64\,(D^4 - d^4)}{D/2}$$

$$= \frac{\pi}{32D}(D^4 - d^4)$$

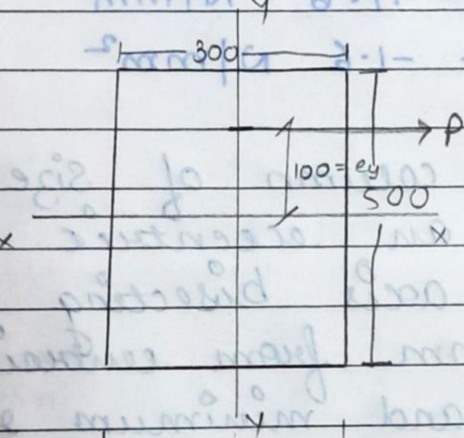
$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$e_y = \frac{\frac{\pi}{32D}(D^4 - d^4)}{\pi/4\,(D^2 - d^2)} = \frac{1}{8D}\frac{(D^2+d^2)(D^2-d^2)}{(D^2-d^2)}$$

$$e_y = \frac{D^2 + d^2}{8D} = e_x$$

Examples :—

(1) A rectangular column section of size 30 × 50 cm carries an eccentric load of 1200 KN at an eccentricity of 10 cm from its C.G on the ~~axis~~ axis bisecting 30 cm side determine maximum and minimum stresses induced in the section Draw stress distribution diagram.



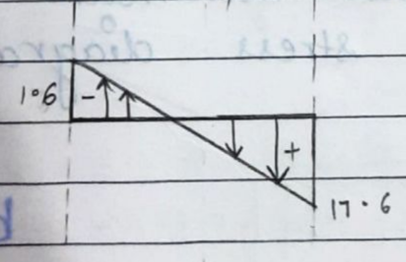
~~$A = 300 \times 500\ m$~~

$b = 300\ mm$

$d = 500\ mm$

$P = 1200 \times 10^3\ N$

$e_y = 100$



$A = b \times d$

$= 300 \times 500 = 150000\ mm^2$

$$\frac{P}{A} = \frac{1200 \times 10^3}{150000} = 8$$

$M = P \times e$

$= 1200 \times 10^3 \times 10 \times 10 = 12 \times 10^7\ Nmm$

$$Z_{xx} = \frac{I_{xx}}{y}$$

$$= \frac{300 \times (500)^3}{12 \times 250} = 12.5 \times 10^6\ mm^3$$

$$\frac{M}{Z} = \frac{12 \times 10^7}{12.5 \times 10^6} = 9.6 \ N/mm^2$$

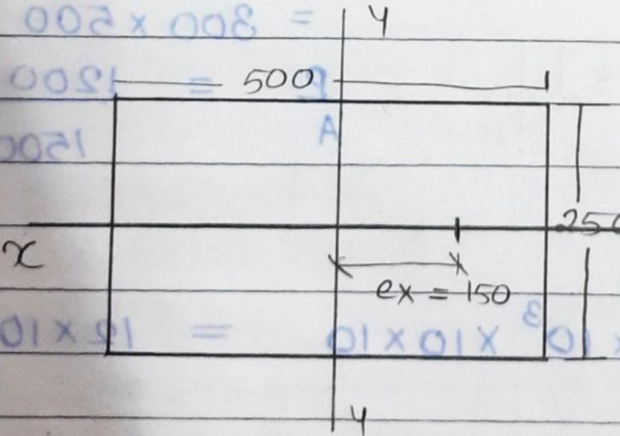
$$\sigma_{\substack{max \\ min}} = \sigma_0 \pm \sigma_b$$

$$= \frac{P}{A} \pm \frac{M}{Z}$$

$$\sigma_{max} = 8 + 9.6 = 17.6 \ N/mm^2$$

$$\sigma_{min} = 8 - 9.6 = -1.6 \ N/mm^2$$

(2) A rectangular column of size 500mm x 250mm carries an eccentric load of 1000 KN on the axis bisecting the thickness at 150 mm from centroidal axis. Find maximum and minimum resultant stress and draw stress diagram



$b = 500$

$d = 250$

$P = 1000 \times 10^3 \ N$

$e_x = 150$

$A = 500 \times 250$

$= 125000$

$$\frac{P}{A} = \frac{1000 \times 10^3}{125000} = 8$$

$M = P \times e$

$= 1000 \times 10^3 \times 150$

$= 15 \times 10^7$ Nmm.

$Z_{yy} = \dfrac{I_{yy}}{y}$

$= \dfrac{\frac{db^3}{12}}{y} = \dfrac{250 \times (500)^3}{12 \times 250}$

$= 10.4 \times 10^6 \ mm^3$

$\dfrac{M}{Z} = \dfrac{15 \times 10^7}{10.4 \times 10^6} = 14.42$

$\sigma_{\substack{max \\ min}} = \sigma_0 \pm \sigma_b$

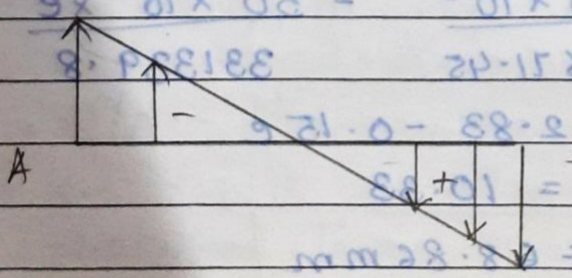
$= \dfrac{P}{A} \pm \dfrac{M}{Z}$

$\sigma_{max} = 8 + 14.42$

$= 22.42 \ N/mm^2$

$\sigma_{min} = 8 - 14.42$

$= -6.42 \ N/mm^2$



$\varepsilon_{max} = 22.4 \ N/mm^2$

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(3) A cast iron column having 150mm ϕ carries eccentric load of 50 KN. If max. tensile stress is not to exceed $7.5 N/mm^2$. find permissible eccentricity of load on column.

$d = 150 mm$

$p = 50 KN = 50 \times 10^3 N$

$\sigma_{min} = 7.5 N/mm^2$ (tensile)

$A = \frac{\pi}{4} \times d^2$

$= \frac{\pi}{4} \times 150^2$

$= 17671.45 mm^2$

$\therefore \frac{P}{A} = \frac{50 \times 10^3}{17671.45} = 2.83$

$Z_{YY} = \frac{I_{YY}}{Y} = \frac{\pi/64 \times d^4}{75} = 331339.8$

$\sigma_{min} = \frac{P}{A} - \frac{M}{Z}$

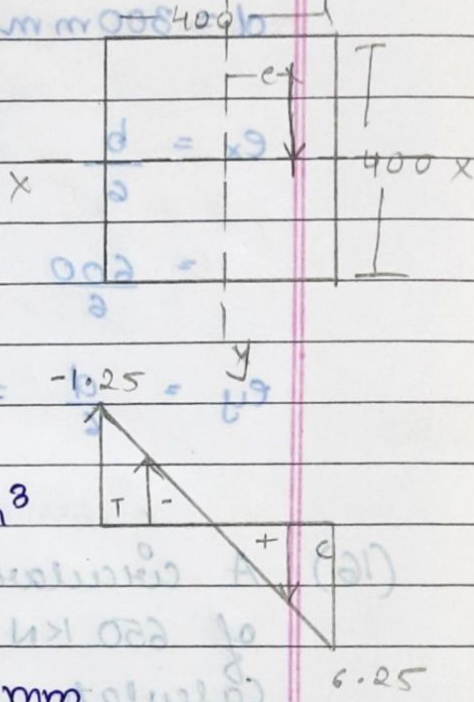
$-7.5 = \frac{80 \times 10^3}{17671.45} - \frac{50 \times 10^3 \times e}{331339.8}$

$\therefore -7.5 = 2.83 - 0.15 e$

$0.15e = 10.33$

$\boxed{e = 68.86 mm}$

(5) A square column of 400 mm side carrying a compressive load of 400 KN at an eccentricity of 100 mm on x-x axis. Find σ_{max} and σ_{min}.



$A = 400 \times 400 = 160000\ mm^2$

$P = 400\ KN = 400 \times 10^3\ N$

$e = 100\ mm$

$y = 200\ mm$

$\therefore \frac{P}{A} = \frac{400 \times 10^3}{160000} = 2.5\ N/mm^2$

$Z_{yy} = \frac{I_{yy}}{y} = \frac{400 \times 400^3}{12 \times 200}$

$= 10.66 \times 10^6\ mm^3$

$M = P \times e$

$= 400 \times 10^3 \times 100 = 4 \times 10^7\ Nmm$

$\therefore \frac{M}{Z} = \frac{4 \times 10^7}{10.66 \times 10^6} = 3.75$

$\sigma_{max} = \frac{P}{A} + \frac{M}{Z}$

$= 2.5 + 3.75$

$= 6.25\ N/mm^2$ ... compressive

$\sigma_{min} = P/A - M/Z$

$= 2.5 - 3.75$

$= -1.25\ N/mm^2$ Tensile

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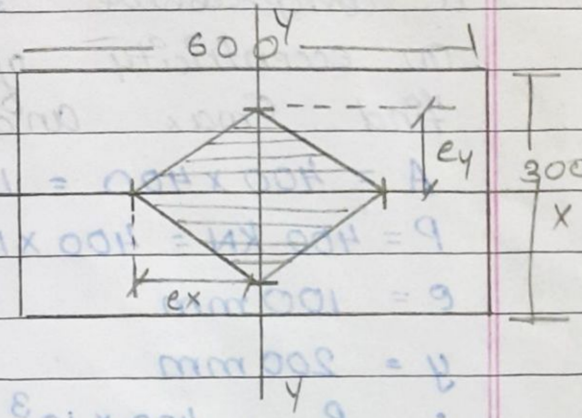
(13) Draw core of rectangular section of size 600mm x 300



b = 600mm

d = 300mm

$e_x = \frac{b}{6}$

$= \frac{600}{6} = 100$ mm

$e_y = \frac{d}{6} = \frac{300}{6} = 50$ mm

(16) A circular column 400 mm ϕ carries load of 650 KN at an eccentricity of 100 mm. Calculate max. and min. stress for column section.

d = 400mm

y = 200 mm

P = 650 KN = 650×10^3 N

e = 100 mm

$A = \pi/4 \times d^2 = \pi/4 \times 400^2$

$= 125663.70$ mm^2

$\therefore \frac{P}{A} = \frac{650 \times 10^3}{125663.70} = 5.17$

$$Z_{yy} = \frac{I_{yy}}{y} = \frac{\pi/64 \times d^4}{200} = \frac{\pi/64 \times 400^4}{200}$$

$$= 6.28 \times 10^6 \ mm^3$$

$$M = P \times e = 650 \times 10^3 \times 100 = 65 \times 10^6 \ Nmm$$

$$\therefore \frac{M}{Z} = \frac{65 \times 10^6}{6.28 \times 10^6} = 10.35$$

$$\sigma_{max} = \frac{P}{A} + \frac{M}{Z}$$

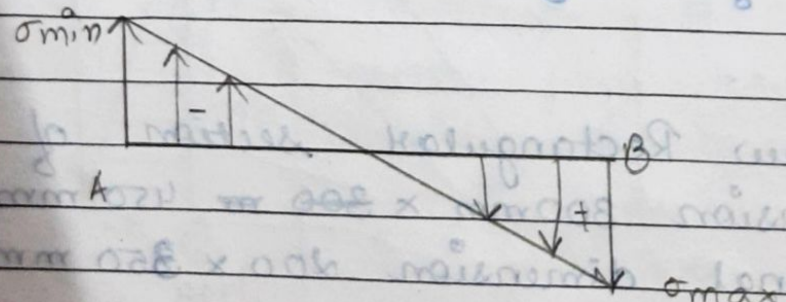
$$= 5.17 + 10.35$$

$$= 15.52 \ N/mm^2 \ \ldots \text{compressive}$$

$$\sigma_{min} = \frac{P}{A} - \frac{M}{Z}$$

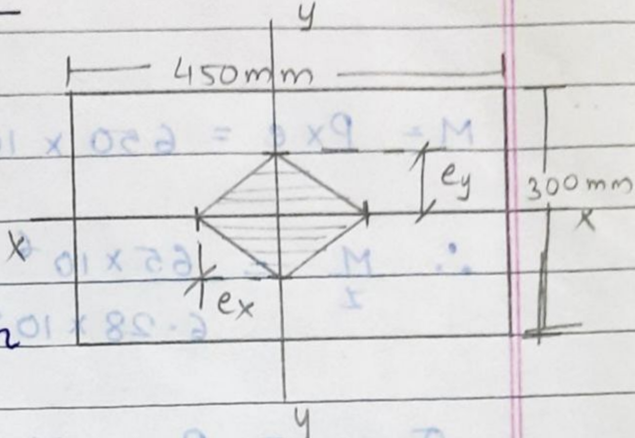
$$= 5.17 - 10.35$$

$$= -5.18 \ N/mm^2 \ \ldots \text{tensile}$$



(20) Draw core section for following :—

a. Rectangular section :—



$b = 450\ mm$

$d = 300\ mm$

$e_x = \frac{b}{6} = \frac{450}{6} = 75\ mm$

$e_y = \frac{d}{6} = \frac{300}{6} = 50\ mm$

b. Hollow Circular 200mm ϕ.

$d = 200\ mm$

$e_x = \frac{d}{8} = \frac{200}{8} = 25\ mm$

$e_y = \frac{d}{8} = \frac{200}{8} = 25\ mm$

c. Hollow Rectangular section of external dimension 300mm × ~~300 m~~ 450 mm and internal dimension 200 × 350 mm.

$BD = 300 \times 460\ mm$

$bd = 200 \times 350\ mm$

$$e_x = \frac{DB^3 - db^3}{6B(BD - bd)}$$

$$= \frac{9.35 \times 10^9}{1.17 \times 10^8}$$

$$e_x = 79.90\ mm$$

$$e_y = \frac{BD^3 - bd^3}{6D(BD - bd)} = \frac{1.87 \times 10^{10}}{1.75 \times 10^8}$$

$$= e_y = 106.85\ mm$$

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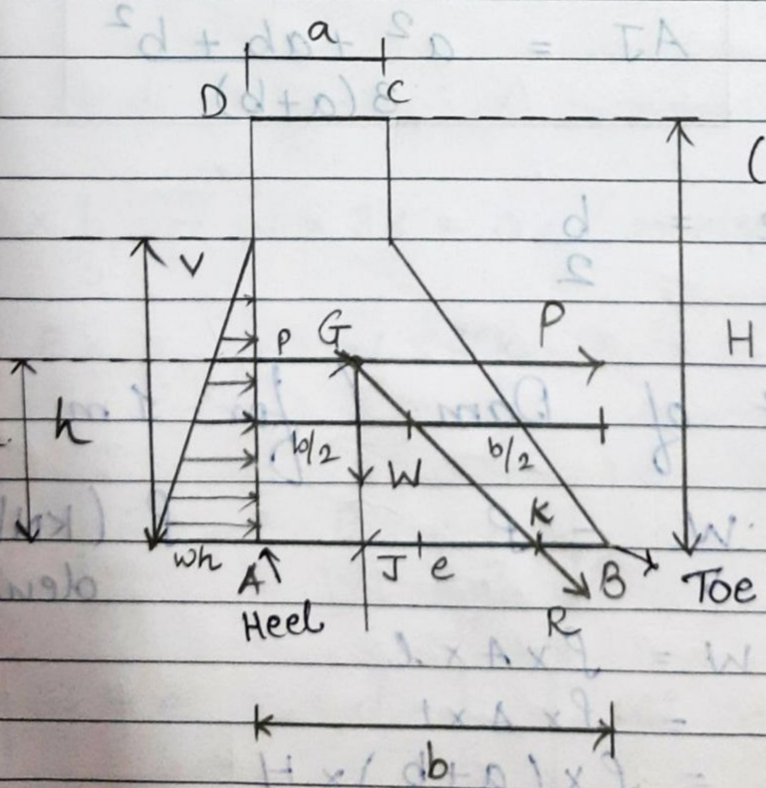
• Dam And Retaining Wall :—



• $\omega = 1000 kg/m^3$ (density of H_2O)

where,

a = top width
b = bottom width
H = total height of dam
h = total height of water
W = Weight of dam
P = Pressure force
R = Resultant force

∴ Distance of CG from extreme fibre ...

$$\boxed{AJ = \frac{a^2 + ab + b^2}{3(a+b)}}$$

$$\Rightarrow Ae = \frac{b}{2}$$

(1) Weight of Dam (for 1 m length)

$$\frac{W}{V} = \rho$$

ρ (kN/m³) ⇒ density of material of dam.

$$W = \rho \times A \times l$$
$$= \rho \times A \times 1$$

$$\boxed{W = \rho \times \left(\frac{a+b}{2}\right) \times H}$$

(II) Water Pressure

$$P = \frac{1}{2} \times \omega h \times h$$

$$\boxed{P = \frac{\omega h^2}{2}}$$

ω = density of water

(III) Resultant force

$$\boxed{R = \sqrt{(W)^2 + (P)^2}}$$

A B

$eK = e$

$P \times \frac{h}{3} - W \times JK = 0$ (moment @ K)

$$P \times \frac{h}{3} = W \times JK$$

$$\boxed{JK = \frac{P}{W} \times \frac{h}{3}}$$

$\therefore$ $\boxed{eK = e = d - \frac{b}{2}}$ $\because$ $d = AJ + JK$

$\because$ e = eccentricity of dam

- d should be $\leq \frac{2}{3} b$

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$$\sigma_{\substack{max \\ min}} = \left[\frac{W}{b} \left(1 \pm \frac{6e}{b}\right) \right]$$

- Retaining Wall

A retaining wall is a structure used to retain soil. The basic difference b/w dam and retaining wall is that a dam retains water and retaining wall retains earth (soil). So while analysis in dam structure we consider water pressure and in analysis of retaining wall we consider earth pressure.

Earth pressure =

$$P = \frac{wh^2}{2} \times K_a$$

where

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$\because$ K_a = active coefficient of earth pressure

ϕ = angle of repose.

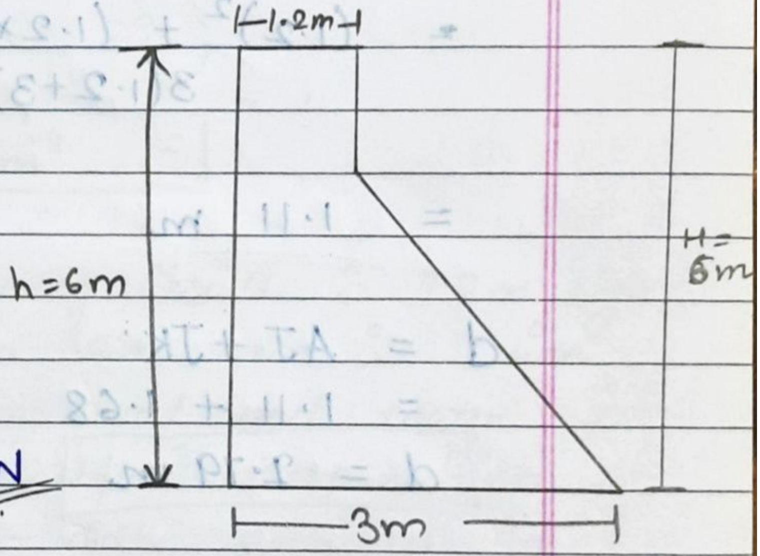
(1) A masonary dam 6m height, 3m wide, at base and 1·2m wide at top. retains water ~~of~~ on vertical face for full height considering density of masonary as 17 KN/m³ and density of water as 10 KN/m³. Find out maximum and minimum pressure intensities at the base.

$H = 6m$

$h = 6m$

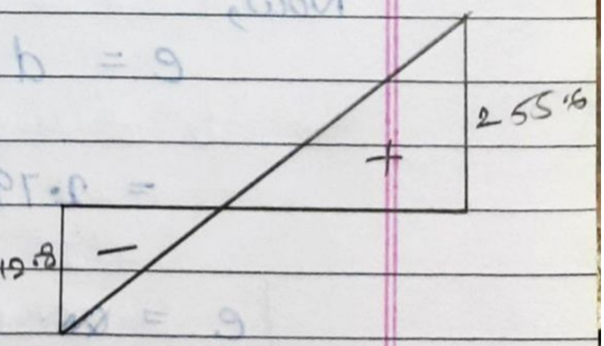
$\rho = 17 KN/m^3$

$w = 10 KN/m^3$



(1) $P = \dfrac{wh^2}{2}$

$= \dfrac{10 \times 6^2}{2} = \underline{180 KN}$



(11) $W = \rho \times V$

$= \rho \times A$

$= \rho \times \left(\dfrac{a+b}{2}\right) \times H$

$= 17 \times \left(\dfrac{1.2+3}{2}\right) \times 6$

$= \underline{214.2}$ KN

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$$JK = \frac{P}{W} \times \frac{h}{3}$$

$$= \frac{180}{214.2} \times \frac{6}{3} \Rightarrow 1.68\,m$$

$$AJ = \frac{a^2 + ab + b^2}{3(a+b)}$$

$$= \frac{(1.2)^2 + (1.2 \times 3) + (3)^2}{3(1.2+3)}$$

$$= 1.11\ m$$

$$d = AJ + JK$$

$$= 1.11 + 1.68$$

$$\boxed{d = 2.79\ m}$$

Now,

$$e = d - \frac{b}{2}$$

$$= 2.79 - \frac{3}{2}$$

$$\boxed{e = 1.29\,m}$$

Eg - 2, 3, 4, 6, 7, 8, 9, 10, 11

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$$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{214.2}{3}\left(1 + \frac{6 \times 1.29}{3}\right)$$

$$\boxed{\sigma_{max} = 255.61 \ KN/m^2}$$

$$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{214.2}{3}\left(1 - \frac{6 \times 1.29}{3}\right)$$

$$\boxed{\sigma_{min} = -112.81 \ KN/m^2.}$$

(4) A rectangular retaining wall is 7.2 m high retains water upto 6 m on its side. Density of wall material and water is 23.5 KN/m³ and 10 KN/m³ resp. Find out minimum base width required to avoid tension at base.

$$\rho_{mat} = 23.5 \ KN/m^3$$
$$\rho_w = 10 \ KN/m^3$$

$$P = \frac{wh^2}{2} = \frac{10 \times 6^2}{2} = 180 \ KN$$

$$W = A \times \rho$$
$$= b \times 7.2 \times 23.5$$
$$= 169.2b$$

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$$JK = \frac{P}{W} \times \frac{h}{3}$$

$$= \frac{180}{169.2b} \times \frac{6}{3} = \frac{2.12}{b} \quad \text{--- I.}$$

$$d = AJ + JK$$

$$= \frac{b}{2} + \frac{2.12}{b}$$

Now,

$$d = \frac{2}{3} b$$

$$\frac{b}{2} + \frac{2.12}{b} = \frac{2b}{3}$$

$$\frac{b^2 + (2.12 \times 2)}{2b} = \frac{2b}{3}$$

$$b^2 + 4.24 = \frac{2b \times 2b}{3}$$

$$b^2 + 4.24 = \frac{4b^2}{3}$$

$$b^2 + 4.24 = 1.3 b^2$$

$$4.24 = 1.3 b^2 - b^2$$

$$4.24 = 0.3 b^2$$

$$\frac{4.24}{0.3} = b^2$$

$$b^2 = 14.13$$

$$b = \sqrt{14.13}$$

$$\therefore \quad b = 3.76 m$$

A rectangular retaining wall section is 2m wide, it retains water upto full height. Find out minimum height required for following.

- (i) when it is just at the point of overturning
- (ii) when it is just at the point of sliding
- (iii) If no tensile stress is produced at the base of the section.

The density of wall material and water is 22 KN/m³ and 10 KN/m³ resp. Take coefficient of friction as 0.5.

Case – (i) $P \times \frac{h}{3} = W \times \frac{b}{2}$

$W = A \times \rho$

$= b \times H \times 22 = 2 \times H \times 22$

$W = 44H$

$P = \frac{\omega H^2}{2}$

$= \frac{10}{2} \times H^2 = 5H^2$

$\therefore 5H^2 \times \frac{H}{3} = 44H \times \frac{2}{2}$

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$$\frac{5H^3}{3} = 44\,H$$

$$5H^2 = 44 \times 3$$

$$H^2 = \frac{44 \times 3}{5}$$

$$H^2 = 26.4m$$

$$H = \sqrt{26.4m}$$

$$\therefore \boxed{H = 5.1m}$$

Case - (ii)

$$P = 5H^2$$

$$W = 44\,H$$

$$\mu = 0.5$$

$$P = \mu\,W$$

$$5H^2 = 0.5 \times 44\,H$$

$$5H = 0.5 \times 44$$

$$H = \frac{0.5 \times 44}{5}$$

$$\therefore \boxed{H = 4.4\,m}$$

Case - (iii) $d \leq \frac{2}{3} b$

$$JK = \frac{P}{W} \times \frac{h}{3}$$

$$= \frac{5H^2}{44\cancel{H}} \times \frac{\cancel{H}}{3}$$

$$= 0.03\ H^2$$

$$AJ = \frac{b}{2} = 1$$

$$d = AJ + JK$$

$$= 1 + 0.03\ H^2$$

$$d = \frac{2}{3} b$$

$$1 + 0.03\ H^2 = \frac{2}{3} \times 2$$

$$1 + 0.03\ H^2 = \frac{4}{3}$$

$$0.03\ H^2 = 1.3 - 1$$

$$0.03\ H^2 = 0.3$$

$$H^2 = \frac{0.3}{0.03}$$

$$H^2 = 10$$

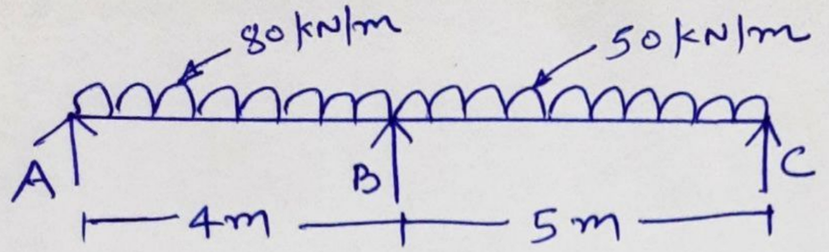
$$\therefore \boxed{H = 3.16\ m}$$

22/3/21

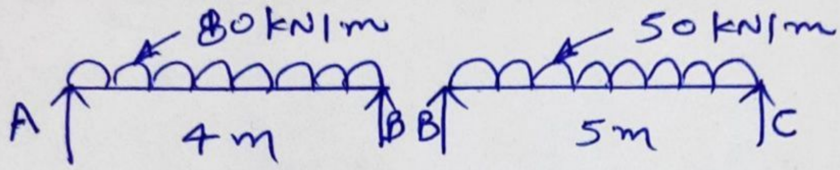
ex:3

~~SLOPE~~ CONTINUOUS BEAM

Draw
S.F.D
B.M.D.

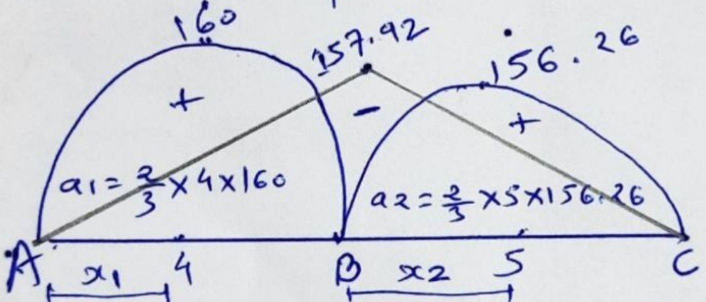


Sol^n:



$$M = \frac{wl^2}{8} = \frac{80 \times 4^2}{8} = 160 \text{ kN.m}$$

$$M = \frac{wl^2}{8} = \frac{50 \times 5^2}{8} = 156.26 \text{ kN.m}$$



Apply theorem of three Moment

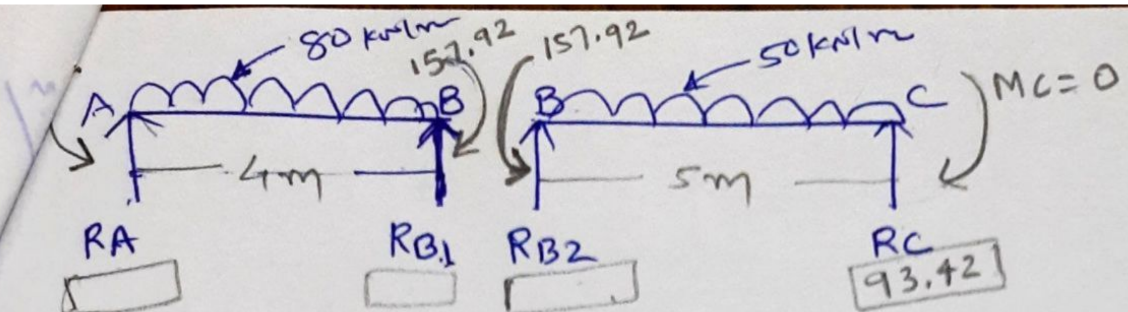
$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -\frac{6a_1x_1}{l_1} - \frac{6a_2x_2}{l_2}.$$

$$0 + 2M_B(4+5) + 0 = -\frac{6\left(\frac{2}{3}\times4\times160\right)(2)}{4} - \frac{6\left(\frac{2}{3}\times5\times156.26\right)\times2.5}{5}$$

$$18M_B = -1280 - 1562.5$$

$$18M_B = -2842.5$$

$$\boxed{M_B = -157.92 \text{ kN.m}}$$



Take Moment @ A

$(R_{B_1} \times 4) + 0 = \left(80 \times 4 \times \frac{4}{2}\right) + 157.92$

$R_{B_1} = 199.48\,kN$

$R_A = (80 \times 4) - 199.48$

$= 120.52\,kN$

Take Moment @ B

$(R_C \times 5) + 157.92 = (50 \times 5 \times 2.5) \times 0$

$R_C = 93.42\,kN$

$R_{B2} = (50 \times 5) - 93.42$

$= 156.58\,kN$

$R_B = R_{B1} + R_{B2}$

$= 199.48 + 156.58 = \boxed{356.06\,kN}$

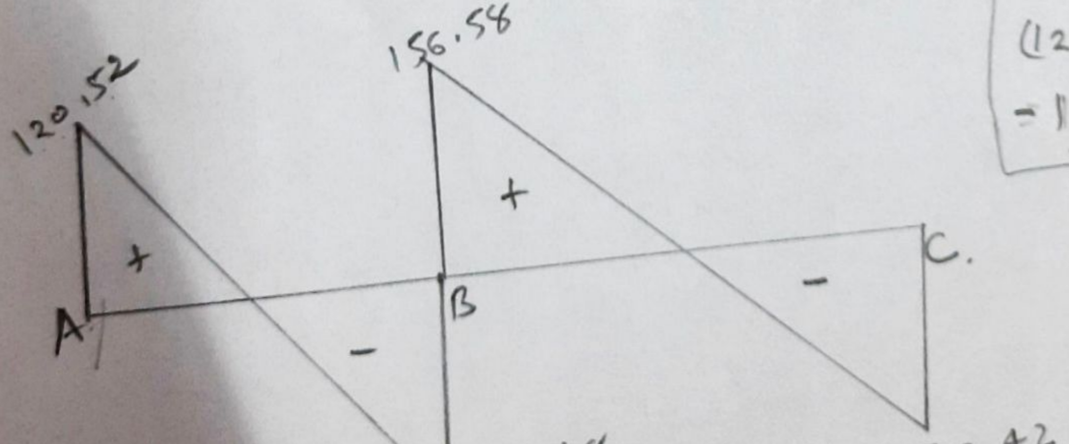
$R_A = 120.52\,kN$, $R_B = 356.06\,kN$, $R_C = 93.42\,kN$

cross check udl

$120 - (80 \times 4)$

$(120 - 320)$

$= 199.48$



S.F.D.

Ex: 7 H.W.